

Протров, Уматов, Задан по тер. верахт.

Зубов, Себастьянов, сборник задач по тер. верахт.
мечен

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$(\Omega, \mathcal{A}, P)$ - в.п., $\Omega \neq \emptyset$ - м-во эл-х исходов (эл. событий)

$$A \cap B = AB$$

$$A \cup B = A \bar{B}, \bar{B} = \Omega \setminus B$$

$\mathcal{A}$ - класс под-в Ω , над. к-ми, над. к-ми, на к-х определены вероят-ти (σ-алгебра)

P - вер-ть: 1) $P(A) \geq 0 \quad \forall A \in \mathcal{A}$
 2) $P(\Omega) = 1$
 3) $\forall A, B \in \mathcal{A} \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$AB = \emptyset \Leftrightarrow A, B$ - несовместн.

Класс. опред-ие вер-ти

1) $|\Omega| < \infty$, $\Omega = \{\omega_1, \dots, \omega_n\}$, $|\Omega| = n \quad \exists p_1, \dots, p_n \geq 0, \sum_{i=1}^n p_i = 1 \quad P(\omega_i) = p_i, i=1, \dots, n$
 $\forall A \in \mathcal{A} \quad P(A) := \sum_{i: \omega_i \in A} p_i$ (p_i - все равны в классич. подходе)

2) $p_i = \frac{1}{n}$
 3) $P(A) = \frac{|A|}{|\Omega|}$

$$A_n^k = \frac{n!}{k!(n-k)!} \quad C_n^k = \frac{n!}{k!(n-k)!}$$

без повторений

Д/З. А и В несовместны
 Из 1), 2) $\Rightarrow$ 3).

$$n_1 + n_2 + \dots + n_k = n, \quad C_n^{n_1} \cdot C_{n-n_1}^{n_2} \cdot C_{n-n_1-n_2}^{n_3} \cdot \dots \cdot C_{n-n_1-\dots-n_{k-1}}^{n_k} = \frac{n!}{n_1! \cdot \dots \cdot n_k!}$$

n шаров к-х k групп (числ. перестановки групп).

Д/З:
 ЗЧЧ
 1.2
 1.5
 1.8
 1.9
 1.26
 1.30
 1.41-1.45
 + ПУ
 1.28
 1.38
 1.41-1.43
 1.48
 1.59

§ 1.2

$26! = |\Omega|$, $A = \{4 \text{ туза}\}$ $|A| = 33 \cdot 4! \cdot 32!$
 арифм.: $15 \cdot 4! \cdot 32!$

§ 1.3

$|\Omega| = 3!$

§ 1.5 $|\Omega| = 3^3$, $\Omega = \{(\omega_1, \omega_2, \omega_3) : \omega_i \in \{0, 1, 2\}\}$

§ 1.8 а) $P = \frac{1}{3}$ б) в) $P(C) =$

§ 1.28

$A = \{ \geq 1, "6" \text{ при } 4^x \text{ бросках } 6 \text{ кубов} \} \quad 1 - \left(\frac{5}{6}\right)^4$
 $B = \{ \geq 1, "(6, 6)" \text{ при } 24^x \text{ бросках парн кубов} \} \quad 1 - \left(\frac{35}{36}\right)^4$

Д/р: ЗЧЧ

§ 1.26

Равномерное распределение

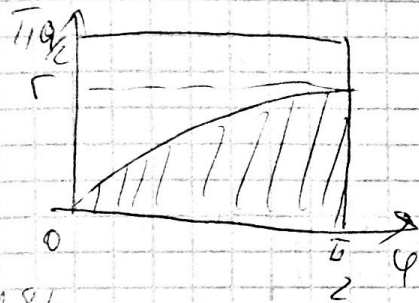
Geom. вероятн.;

$\Omega, \mu(\Omega) < \infty; A \subseteq \Omega, \mu(A) > 0$, тогда $P(A) = \frac{\mu(A)}{\mu(\Omega)}$ без учета нормировки.
 A в geom. вероятн.

Пр. 1.84

$(x, y); x, y \in [0, 1]; \Omega = \{(x, y) | x, y \in [0, 1]\}$
 $A = \{(x, y) \in \Omega | |x - y| \leq \frac{1}{6}\}$  $P(A) = \frac{11}{36}$

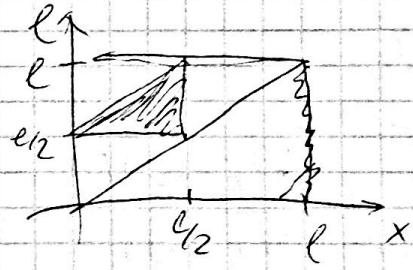
$\Omega = \{(\varphi, x) | 0 \leq \varphi < \pi, 0 \leq x \leq \frac{\pi}{2}\}$, площадь
 $\frac{\pi^2}{4}$; $x \leq \sin \varphi$; $A = \{(\varphi, x) : x \leq \sin \varphi\}$



$\mu(\Omega) = \frac{\pi^2}{4}$; $x = r \sin \varphi$; $P(A) = \frac{-r \cos \varphi \big|_0^{\pi/2}}{\frac{\pi^2}{4}} = \frac{4}{\pi^2} \approx \frac{4}{9.86} \approx \frac{4}{10} = \frac{2}{5}$

Пр. 1.86

$\Omega = \{(x, y) | 0 \leq x \leq l, 0 \leq y \leq l, x < y\}$
 $|x - y| < x + y \leq l - y$
 $x < l - y$
 $x < (y - x) + (l - y)$
 $(y - x) < x + (l - y)$
 $(l - y) < x + (y - x)$
 $x < l - x$
 $2y - l < 2x$
 $l - y < x$

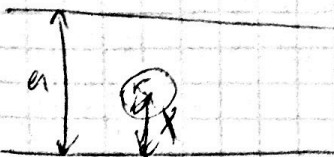


$\begin{cases} 2x < l \\ 2x > 2y - l > 0 \\ x < l - y < l \end{cases} \quad \begin{cases} x < l/2 \\ y > l/2 \\ x > y - l/2 \end{cases}$

$P(A) = \frac{1}{4}$

518
1744
1.92
1.88
1.96
1.88
1.93
304
1.64
1.65
1.70
1.71

Пр. 1.92



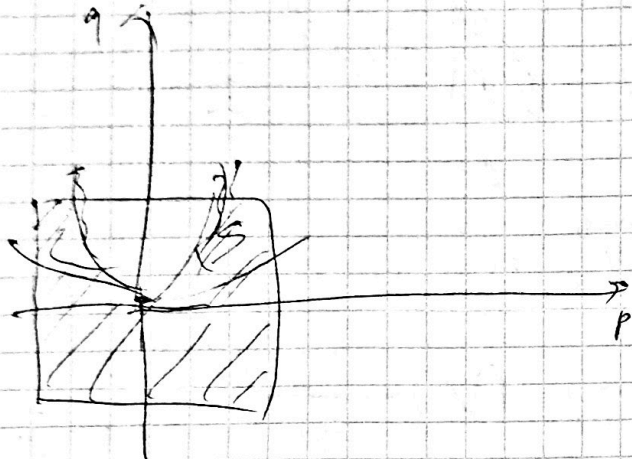
$R \subset \mathbb{R}^2; \Omega = \{x | x \in [0, a]\}, A = \{x | x \in [0, R]\}$

~~Пр. 1.93~~

~~Задача 1.98~~ $0 < a < R$
 $(1 - (\frac{a}{R})^2)^N$

2/p. 1100
1.93

$[1; 1]$; p, q - коэф. ; $x^2 + px + q = 0$, $D = p^2 - 4q \geq 0$ $\left| \begin{array}{l} x_{1,2} = \frac{-p \pm \sqrt{D}}{2} \\ x_1 + x_2 = -p \\ x_1 \cdot x_2 = q \end{array} \right.$



$4q \leq p^2$, $q \leq \frac{p^2}{4}$; $-p \pm \sqrt{D} = \frac{m}{n}$
 $\int_1^p \frac{x^2}{4} dx + \frac{1}{2} \in \mathbb{Q}$

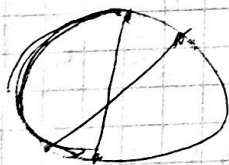
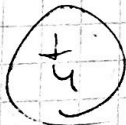
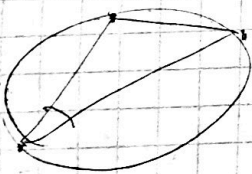
1.96

n т.; m раз. на a_1 . $\left(\frac{a_1}{a_1+a_2}\right)^m \cdot \left(\frac{a_2}{a_1+a_2}\right)^{n-m}$

1.98

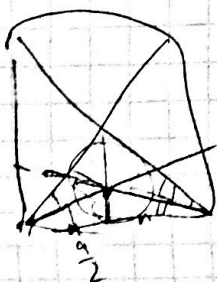
1.103

$\frac{|(0, 90)|}{|(0, 180)|} = \left(\frac{1}{2}\right)$



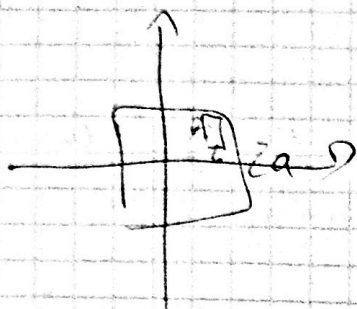
304

1.102



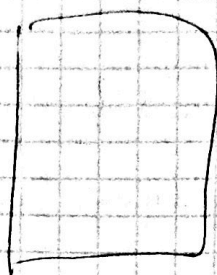
$\frac{1}{4} \log \frac{45}{2} = \frac{h}{\frac{19}{2}}$ $\Rightarrow h = \frac{9}{2} \log \frac{45}{2} \Rightarrow p = \frac{1 \cdot \frac{1}{2} \cdot \frac{1}{2} \log \frac{45}{2}}{2}$
 $= \log \frac{45}{2}$

2.65



$$\frac{1}{3}(x)$$

2.67



$$\frac{1}{3}x^3 - \xi_1^2 x + \xi_2 =$$

$$= \frac{1}{3}(x - a_1)(x^2 + a_3x + a_4)$$

$$\Delta < 0 \Rightarrow a_3^2 - 4a_2a_4 < 0$$

$$\frac{1}{3}(a_2x^3 + x^2(a_3 - a_1a_2) + x(a_4 - a_1a_3) - a_1a_4)$$

$$\begin{cases} a_2 = 1, a_3 = a_1 \\ a_3 - a_1a_2 = 0, a_4 = a_1 \\ a_4 - a_1a_3 = -\xi_1^2, a_4 - a_1^2 = -\xi_1^2 \\ -a_1a_4 = \xi_2 \\ a_3a_4 = \xi_2 \\ a_3^2 - 4a_4 < 0 \end{cases}$$

$$\begin{cases} a_3^2 - 4a_4 < 0, a_4 = \frac{a_3^2}{4} \\ a_3a_4 = \xi_2 \\ a_3^2 - a_4 = \xi_1^2 \end{cases}$$

$$a_4 = \frac{a_3^2}{4} - \xi_1^2$$

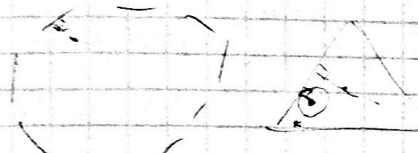
$$a_3^2 - a_3\xi_1^2 - \xi_2^2 = 0$$

$$\frac{1}{4}a_3^4 - 8a_3^2a_4 + 16a_4^2 < 0$$

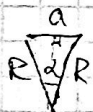
$$a_3^4 - 2a_3^2a_4 + a_4^2 = \xi_1^2$$

2.68

вспомог. со ст. ξ_i ; не является
матр. поз. Γ .



$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$



$$\alpha = \frac{360}{k}$$

$$a^2 = 2R^2 - 2R^2 \cos \alpha$$

$$2R^2(1 - \cos \alpha) = a^2$$

$$R = \frac{a}{\sqrt{2(1 - \cos \alpha)}}$$

$$\cos \frac{\alpha}{2} = \frac{r}{R} \Rightarrow r = R \cos \frac{\alpha}{2} = \frac{a \cos \frac{\alpha}{2}}{\sqrt{2(1 - \cos \alpha)}}$$

$$\sqrt{1 - \cos \alpha} = \sqrt{2 \sin^2 \frac{\alpha}{2}}$$

$$\Rightarrow \frac{a \cos \frac{\alpha}{2}}{2 \sin \frac{\alpha}{2}} = \frac{a}{2} \cotg \frac{\alpha}{2} =$$

$$= \frac{a}{2} \cotg \left(\frac{360}{2k} \right)$$

New Seminar

$$P(A|B) = \frac{P(AB)}{P(B)} = P(A), \text{ если независ.}$$

1) $P(A) \in \{0, 1\} \Rightarrow A, B$ - нез. $\forall B$.

2) $A \cup B$ независ. $\Leftrightarrow \bar{A} \bar{B}$ независ. $\Leftrightarrow \dots$

3) A, A - нез. $\Rightarrow P(A) \in \{0, 1\}$.

4) $\exists P(A)P(B) > 0$, тогда A, B - нез. $\Rightarrow$
 $\Rightarrow P(AB) > 0 \Rightarrow AB \neq \emptyset$

$$A \cap B = \emptyset \Rightarrow P(A \cap B) = 0 \neq P(A) \cdot P(B) \Rightarrow A, B \text{ не независимы}$$

5) $P(A|B)$ - вероятность $\Rightarrow \dots$

$A = \{\text{на } I-\delta\}$, $B = \{\text{на } I-\delta\}$, $C = \{\text{ровно, на } I-\delta \text{ или } II-\delta\}$.
 Пусть A, B, C независимы в совокупности, если верно, если же
 не так, то не так.

Пусть $A_1, \dots, A_n$ независимы $\Leftrightarrow \forall 2 \leq i \leq n \quad \forall 1 \leq i_1, i_2 < \dots < i_k \leq n \quad P(\bigcap_{j=1}^k A_{i_j}) = \prod_{j=1}^k P(A_{i_j})$

Пусть $A_1, \dots, A_n$ независимы в совокупности $\Leftrightarrow$

$$P(A_1^{\delta_1} \dots A_n^{\delta_n}) = P(A_1^{\delta_1}) \dots P(A_n^{\delta_n}) \quad \forall \delta_1, \dots, \delta_n \in \{0, 1\}, \text{ где } A_i^{\delta_i} = \begin{cases} A_i, & \delta_i = 1 \\ \bar{A}_i, & \delta_i = 0 \end{cases}$$

Получим: $P(A_1, \dots, A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1, A_2)P(\dots) \times \dots \times P(A_n|A_1, \dots, A_{n-1})$

$$P(A_1, \dots, A_n) = \sum_{i=1}^n P(A_i) - \sum_{i < j} P(A_i, A_j) + \sum_{i < j < k} P(A_i, A_j, A_k) - \dots + (-1)^{k+1} \sum_{i_1 < i_2 < \dots < i_k} P(A_{i_1}, \dots, A_{i_k}) + \dots \quad (k = \overline{1, n})$$

Пусть $A_1, \dots, A_n$ независимы $\Rightarrow P(A_1, \dots, A_n) = 1 - P(\bigcap_{i=1}^n \bar{A}_i) = 1 - \prod_{i=1}^n (1 - P(A_i))$

$$P(A_1, \dots, A_n | C) = P(A_1 | C) P(A_2 | A_1, C) \dots P(A_n | A_1, \dots, A_{n-1}, C)$$

304
27
210
212-214
216
217
218
219
225
227
230

5224

a) $B = \{\text{усп. брак}\}$
 $A_k = \{\text{усп. брак } k\text{-го кв.}\}$; $\alpha_k = P(A_k | B)$
 $\beta_k = P(\bar{A}_k | \bar{B})$
 $P(A_1, A_2 | B) = ? = P(A_1 | B) \cdot P(A_2 | B) = \alpha_1 \cdot \alpha_2$

б) $P(\bar{A}_1, \bar{A}_2 | \bar{B}) = P(\overline{A_1 \cap A_2} | \bar{B}) = 1 - P(A_1 \cap A_2 | \bar{B}) = 1 - (1 - \beta_1)(1 - \beta_2)$

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5210

5212

$P_1(A|B) = \frac{P(A \cap B)}{P(B)}$; а с ; P_1

$P(B_1)$

$$P(A) = \frac{b}{a+b}, P_2 = \frac{b}{a+b}; \quad P_n = \left(\frac{b}{a+b}\right)^{n-1} \frac{a}{a+b}; \quad \Sigma$$

$$P(A_1) = P(A)$$

2.13

$$P(AB) = P(A) \cdot P(B|A); \quad P_b(b_i) = P(A_1) \cup P(\bar{A}_1 \bar{A}_2 A_3) \cup \dots$$

$$P(\bar{A}_1 \bar{A}_2 \dots \bar{A}_{k-1} A_k) = P(\bar{A}_1) \cdot P(\bar{A}_2 \dots \bar{A}_{k-1} A_k | \bar{A}_1) = \dots = P(\bar{A}_1) \cdot \dots \cdot P(\bar{A}_{k-1}) \cdot P(A_k | \bar{A}_1 \dots \bar{A}_{k-1})$$

$$= \left(\frac{b}{a+b}\right)^{k-1} \frac{a}{a+b}; \quad A_i = \begin{cases} \text{success} & \text{if } i \text{ is even} \\ \text{failure} & \text{if } i \text{ is odd} \end{cases}$$

$$P(B_1) = \sum_{k=0}^{\infty} \left(\frac{b}{a+b}\right)^{k-1} \frac{a}{a+b} = \frac{a}{a+b} \cdot \frac{1}{1 - \frac{b}{a+b}} = \frac{a}{a+b} \cdot \frac{1}{1 - \left(\frac{b}{a+b}\right)^2} = \frac{a}{a+b} \cdot \frac{(a+b)^2}{a^2 + 2ab + b^2} = \frac{a+b}{a+b} = 1$$

2.14

$$P(B_1) = P(A_1) + P(\dots)$$

$$C_k^0 \frac{1}{k} + C_k^1 (k-1) \cdot \frac{1}{k} + C_k^2 \cdot \frac{1}{k} \left(\frac{1}{k}\right) + \dots + C_k^{k-2} \left(\frac{1}{k}\right)^{k-2} \left(\frac{1}{k}\right) + C_k^{k-1} \left(\frac{1}{k}\right)^{k-1} \left(\frac{1}{k}\right) =$$

$$= \frac{1}{k} \left(1 + \frac{1}{k}\right)^{k-1} < \frac{1}{2}; \quad \frac{1}{k} \left(1 + \frac{1}{k}\right)^{k-1} < \frac{1}{2}; \quad \frac{1}{k} \left(1 + \frac{1}{k}\right)^{k-1} < \frac{1}{2};$$

$$\frac{1}{k} \left(1 + \frac{1}{k}\right)^{k-1} < \frac{1}{2}; \quad \frac{1}{k} \left(1 + \frac{1}{k}\right)^{k-1} < \frac{1}{2}; \quad \frac{1}{k} \left(1 + \frac{1}{k}\right)^{k-1} < \frac{1}{2};$$

2.16

2.17

2.18

2.25?

$$2k-1 < \frac{1}{2}k^2$$

$$k^2 - 4k + 2 > 0$$

$$\Delta = 16 - 8 = 8$$

$$k_{1,2} = \frac{4 \pm \sqrt{8}}{2}$$

$$= 2 \pm \sqrt{2}$$

$$P(A_1 A_2) = P(A_1) \cdot P(A_2 | A_1); \quad P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$P(A_1 A_2) = P(A_2) \cdot P(A_1 | A_2)$$

2.22

$$P(C) = P(\bar{A}_1 \bar{A}_2 \bar{A}_3 \bar{A}_4 \bar{A}_5)$$

2.28

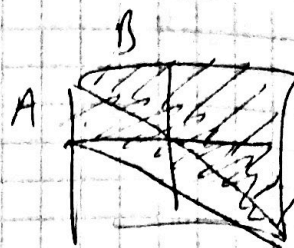
$i=1, \dots, n; p_i; \geq 2 \text{ nodes}; i=1, 2, \dots, n; p_i=0;$

$$\sum_{i=1}^n p_i = p_n$$

$$1 \leq i_1 < i_2 < \dots < i_k \leq n$$

1) A, B нез. $\sum P(A|B|C) = P(A|C) \cdot P(B|C) \quad \forall C \in \mathcal{A}$

$P(A|B|C) \neq P(A|C) \cdot P(B|C) \quad \forall C$



2) $P(AB|C) = P(A|C) \cdot P(B|C) \quad \forall C \stackrel{?}{\Rightarrow} A, B \text{ нез.}$

$C = \Omega; \quad P(AB) = P(A)P(B)$

$\mathcal{P} \cap B$ и $\mathcal{P} B$

$B_1, B_2, \dots$ — ~~независимые~~ $\mathcal{P} \cap C$, т.е.

1) $B_i B_j \neq \emptyset, i \neq j$

2) $\bigcup_i B_i = \Omega$

$P(A) = \sum_k P(A|B_k) P(B_k)$

$P(B_k|A) = \frac{P(A|B_k)}{P(A)} = \frac{P(A|B_k) P(B_k)}{\sum_m P(A|B_m) P(B_m)}$

1: $B = \{ \text{"непр. в"} \}, \bar{B};$

$P(B) = \frac{a}{a+b}, P(\bar{B}) = \frac{b}{a+b}$

2: $A = \{2 \text{ ср. впр}\}$

$P(B|A) = \frac{P(A|B) P(B)}{P(A)} = P(A|B) = \frac{P(A)}{a+b-1} = \frac{a}{a+b-1} \cdot \frac{a-1}{a+b-1} + \frac{b}{a+b-1} \cdot \frac{a}{a+b-1} = \frac{a}{a+b}$

§ 2.43

$B = \{ \text{соедин.} \}, A = \{ \text{прямая соедин.} \}$

B_k — k разовая авария — авария.
двухраз. и 3 раз. авар. авар.

$\in \frac{\lambda^s p e^{-\lambda}}{s!} \sum_{k=s}^{\infty} \frac{(1-p)^{k-s}}{(k-s)!} = \frac{(\lambda p)^s}{s!} e^{-\lambda p}$

$P(C_s) = \sum_{k=s}^{\infty} P(C_s|B_k) \cdot P(B_k) \stackrel{\lambda^k e^{-\lambda}}{\in} \frac{C^s p^s (1-p)^{k-s}}{k! p^s (1-p)^{k-s}}$

$P(C) = \frac{1}{3} = 0.0 + 0.0 + \dots + 0.0;$

$P(H) = 0.0 + 0.0 + \dots + 0.0;$

174
184
704
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2.30

10.
9k.

10.
7up.

~~$P(B|A)$~~ ~~$P(A)$~~

~~$P(A|B) = 2$~~

5/25

2m

curr. noncurr.
over. d.

curr. 28. um 18.

~~5.11.11~~

$$P(A) = \left(\frac{1}{7}\right)^2 \cdot \frac{1}{4} + \left(\frac{2}{7}\right)^2 \cdot \frac{1}{4} + \frac{1}{4}$$

$$a) P = \frac{1}{7} \cdot \frac{2}{7} + \frac{2}{7} \cdot \frac{1}{7} = \frac{2}{7} \cdot \frac{2}{7} = \frac{4}{49}$$

$$= \frac{6}{8 \cdot 7} + \frac{2}{8 \cdot 7} = \frac{8}{8 \cdot 7}$$

2.31

28.
32.

2.31.11 -
again sen. sep.

$$C_5^2 = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4}{2} = 10$$

$$a) P = \frac{C_2^2}{C_5^2} \cdot \frac{1}{4} + \frac{C_2^2}{C_5^2} \cdot \frac{1}{4} + \frac{3 \cdot 2}{C_5^2} \cdot \frac{1}{2} = \frac{1}{10} + \frac{1}{10} + \frac{6}{10} = \frac{8}{10} = \frac{4}{5}$$

2.33

2.34

(1-~~0.1~~) - не брак ; p

2.35

$$P(A) =$$

2.38

$$P(A_1 | A_2 | x_{exp}) = \frac{P(A_1 \cap A_2 | x_{exp})}{P(A_2 | x_{exp})} = \frac{P(A_1 \cap A_2) \cdot P(x_{exp})}{P(A_2) \cdot P(x_{exp})} = \frac{P(A_1 \cap A_2)}{P(A_2)}$$

$$P(\pi | x) =$$

$$P(\pi_{exp} | x) = 0.2$$

$$P(\pi_{bad} | x) = 0.1$$

$$P(\pi | \pi_{exp} \cup \pi_{bad}) = 0.01$$

$$P(\pi | \pi_{bad} | x_{exp}) =$$

$$P(\pi | x) = P(\pi | x_{exp} \cap \pi_{exp}) P(\pi_{exp}) + P(\pi | x_{exp} \cap \pi_{bad}) P(\pi_{bad})$$

дв. д.м. 0.001

Σ 2.42

$$P(A_1|A) = \frac{P(A_1 \cap A|B_1) \cdot P(B_1)}{P(A)}$$

$\frac{11}{33}$

$$P(A|B_1) = \frac{P(A|B_1, A) \cdot P(A)}{P(A|B_1, A) \cdot P(A) + P(A|B_1, \bar{A}) \cdot P(\bar{A})}$$

3 нз. м

$H_1 = \{y \text{ 1 05.м.}\}$ $A = \{65. \text{ мр. } 605 \text{ 610 м.}\}$
 $H_2 = \{y \text{ 1 15.м.}\}$
 $H_3 = \{y \text{ 1 25.м.}\}$

$$P(H_i|A) = \frac{P(A|H_i) \cdot P(H_i)}{P(A|H_1) \cdot P(H_1) + P(A|H_2) \cdot P(H_2) + P(A|H_3) \cdot P(H_3)}$$

$P(A|H_1) = 0$

Суммарно

2.58, 2.59, 2.60, 2.61, 2.62, 2.63, 2.64, 2.65, 2.66, 2.67, 2.71

$P(\xi_n = k) = C_n^k p^k q^{n-k}, k=0, n$

Теллер-уламаса

Схема Пуассона:

Одн. - мр. - бугеке класе

$n \rightarrow \infty, p \text{ фикс } (npq \rightarrow \infty) \Rightarrow$

$\Rightarrow P(X_1 \leq \frac{\xi_n - np}{\sqrt{npq}} \leq X_2) \xrightarrow{D.L.T.} \Phi(X_2) - \Phi(X_1)$

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$

$\Phi(+\infty) = 1$
 $\Phi(0) = \frac{1}{2}$
 $\Phi(-x) = 1 - \Phi(x)$

$\Phi_0(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt = \Phi(x) - \frac{1}{2}$

$\Delta := \sup_x |P(\frac{\xi_n - np}{\sqrt{npq}} \leq x) - \Phi(x)|$

$\sup_{x_1 < x_2} |P(x_1 < \frac{\xi_n - np}{\sqrt{npq}} \leq x_2) - \Phi(x_2) + \Phi(x_1)|$

② Δ

$\Delta \leq \frac{1}{\sqrt{2\pi}} \cdot \frac{p^2 + q^2}{\sqrt{npq}} \quad \forall n \in \mathbb{N}, p \in (0,1)$

0.405... 709999 Const; $p = \frac{1}{2} \Rightarrow \text{const} = 0.3989...$

$\Delta \leq \frac{1}{2\sqrt{npq}}$

$P_n \eta_{n1} \dots \eta_{nn}$

$\xi_n = \sum_{k=1}^n \eta_{n,k} \sim \text{Bin}(n, p_n)$

Пог. (Пуассона, Poisson)

$\exists n \rightarrow \infty, np_n \rightarrow \lambda \in (0, +\infty) \Rightarrow$

$\Rightarrow P(\xi_n = k) \xrightarrow{n \rightarrow \infty} \pi_k := e^{-\lambda} \frac{\lambda^k}{k!}, k=0, 1, n.$

Предельное число утрат в n мн. берем.

Крит. что можно использовать аппрокс.:

$\sum_{k=0}^{\infty} |P(\xi_n = k) - \pi_k| \leq 2np^2 \quad \forall n \in \mathbb{N}, p \in (0,1)$

Σ 2.58

$n=5, p = \frac{11^{0.55}}{20}, q = \frac{9}{20}$

$P(\xi \leq 3) \approx 0.5931$

$P(\xi_n > \frac{n}{2}) \approx 0.99 \quad n \rightarrow \infty?$

n велико, npq → ∞

$$P(\xi_n > \frac{n}{2}) = P\left(\frac{\xi_n - np}{\sqrt{npq}} > \frac{\sqrt{n}(\frac{1}{2} - p)}{\sqrt{pq}}\right) \approx$$

$$\approx \Phi(x) - \Phi(-x) \approx 1 - \Phi\left(\frac{\sqrt{n}(\frac{1}{2} - p)}{\sqrt{pq}}\right) =$$

$$= \Phi\left(\frac{\sqrt{n}(p - \frac{1}{2})}{\sqrt{pq}}\right) = 0.99; \quad \frac{\sqrt{n}(p - \frac{1}{2})}{\sqrt{pq}} \approx 2.3263$$

$$\Rightarrow n = [] + 1 = 586$$

$$npq \geq 586 \cdot 100 \Rightarrow \Delta \leq \frac{1}{2\sqrt{npq}} < 0.1,$$

2.59

$n=1000, p=0.005$ - авар. жм.

$P(\xi_n \leq 3) \approx$; Мухомова-Ланг. не пог.
 $\xi_n^2 \leq \frac{1}{10}$

$$\ominus \sum_{k=0}^3 P(\xi_n = k) \approx \sum_{k=0}^3 e^{-\lambda} \frac{\lambda^k}{k!} \quad \lambda = np_n$$

0.265

2.60

2.60

Вопрос 03 бер. 72:

$$P(\xi_n > 2) =$$

$$= P\left(\frac{\xi_n - np}{\sqrt{npq}} > \frac{2 - np}{\sqrt{npq}}\right) \approx$$

$$= 1 - P\left(\frac{2 - np}{\sqrt{npq}}\right) = \Phi\left(\frac{np - 2}{\sqrt{npq}}\right).$$

$$p = \frac{1}{100}, n = 2000, q = \frac{99}{100}$$

$$\frac{1}{2\sqrt{\frac{99}{100}}} = 0.5025$$

$$P(A) = 1 - \frac{100 \cdot 10^{200}}{10^{200}} =$$

$$= 1 - \frac{100}{99} \left(\frac{99}{100}\right)^{100} = 0.4454$$

2.61

$$p = \frac{1}{2}, a) P\left(\frac{|\xi_n - \frac{n}{2}|}{\sqrt{n \cdot \frac{1}{2} \cdot \frac{1}{2}}} \leq \frac{\sqrt{n}}{2} \cdot \frac{1}{\sqrt{n \cdot \frac{1}{2} \cdot \frac{1}{2}}} = \Phi(1) - \Phi(-1)$$

Уточнение при-ра Сирини:

2.65

Кр. работа

Вероятности событий

$(\Omega, \mathcal{F}, P)$

Ω - произ-ое конечное мн-во

Спр M - класс событий в Ω

$$M = \{A_1, A_2, A_3, \dots\};$$

$\Omega \quad \Omega \quad \Omega$

Например $\Omega = \{0, 1\}$,

$$M = \{ \emptyset, \Omega, \{ \frac{1}{2} \}, \{0, 1\}, \{ \frac{1}{2}, \frac{1}{2} \} \}$$

Спр $\mathcal{F}$ - алгебра событий в Ω :

$$1) \Omega \in \mathcal{F}$$

$$2) A \in \mathcal{F} \Rightarrow \bar{A} = \Omega \setminus A \in \mathcal{F}$$

$$3) A_1, A_2, \dots, A_n \in \mathcal{F} \Rightarrow \bigcup_{i=1}^n A_i \in \mathcal{F}$$

$$\forall A, B \in \mathcal{F}, \text{ то } A \cap B \in \mathcal{F}$$

$$A \setminus B = A \cap \bar{B}.$$

Спр $\mathcal{A}$ - σ -алгебра событий в Ω , если

$$1) \dots$$

$$2) \dots$$

$$3) A_1, A_2, A_3, \dots \in \mathcal{A}, \bigcup_{i=1}^{\infty} A_i \in \mathcal{A}$$

$$\text{Спр } W \in \bigcup_{\alpha} A_{\alpha} \Leftrightarrow \exists \alpha: W \in A_{\alpha}$$

Воп. $\sigma(M)$ (с-алгебра, порожд. множеством M)

- 1) $M \subseteq \sigma(M)$
- 2) $\sigma(M)$ - с-алгебра
- 3) Если $\mathcal{F}$ - с-алгебра и $M \subseteq \mathcal{F}$, то $\sigma(M) \subseteq \mathcal{F}$

1. $\mathcal{A} = \{\Omega, \emptyset\}$ - тривиал. с-алгебра

2. $\mathcal{A} = 2^\Omega$ - все возможные события (полн. с-алгебра)

Борелевская с-алгебра

$$\Omega = \mathbb{R}$$

Опр $\mathcal{B}(\mathbb{R})$ - Борел. с-алгебра - это $\sigma(\{[a, b] : a < b\})$.

$[a, b)$ тоже образует $\mathcal{B}$;

$$\bigcap_{n=1}^{\infty} (a - \frac{1}{n}, b) = [a, b).$$

$$[a, b], [a, b], (-\infty, a), (-\infty, a], (a, +\infty), [a, +\infty), \text{ и т.д.}$$

$$\bigcap_{n=1}^{\infty} (a - \frac{1}{n}, a + \frac{1}{n})$$

$$M = \{A_1, A_2, \dots, A_n\}$$

2^n элементов с-алгебры $\sigma(M)$

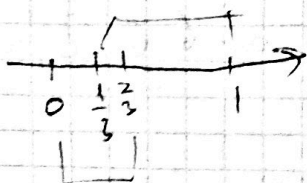
$$A_i \cap A_j = \emptyset, i \neq j$$

$$\bigcup_{i=1}^n A_i = \Omega;$$

Зад. 2.2

$$1) \Omega = [0, 1], M = \{[0, \frac{1}{3}], [\frac{1}{3}, \frac{2}{3}], [\frac{2}{3}, 1]\}$$

$$\sigma(M) = \{[0, 1], \emptyset, [0, \frac{2}{3}], [\frac{2}{3}, 1], [\frac{1}{3}, \frac{2}{3}], [\frac{1}{3}, \frac{1}{3}], [\frac{2}{3}, \frac{2}{3}], [\frac{2}{3}, \frac{1}{3}]\}$$



$$[\frac{1}{3}, \frac{2}{3}], [0, \frac{1}{3}]$$

$$[\frac{2}{3}, 1], [\frac{1}{3}, \frac{2}{3}]$$

А всевозможные разбиения

б) —

2) —

а

Зад. 2.16

$$\Omega = \{0, 1\}$$

$$\mathcal{A} = \{\emptyset, \Omega\}$$

$$A_1, A_2, \dots$$

$$\bigcap_{k=1}^{\infty} A_k \Rightarrow \exists n \in \mathbb{N} : \forall k \geq n \omega \in A_k$$

$$\bigcup_{k=1}^{\infty} A_k \Rightarrow \forall n \in \mathbb{N} \exists k \geq n \omega \in A_k$$

$$\liminf, \limsup, \text{вероятность}$$

Алгебра не с-алгебра

Для работы

$(\Omega, \mathcal{A})$ - у. пр., $\mathcal{A}$ - с-алгебра - это класс событий, замкнутый относительно операций:

- 1) $\Omega \in \mathcal{A}$
- 2) $A \in \mathcal{A} \Rightarrow A^c \in \mathcal{A}$
- 3) $A_1, A_2, \dots \in \mathcal{A} \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{A}$

$$\Omega = \mathbb{R}, \mathcal{B} = \sigma(\{[a, b] : a < b\}) = \sigma(\{(-\infty, x), x \in \mathbb{R}\}) \neq 2^{\mathbb{R}}$$

$$A \cap B = A \cap \bar{B}, \text{ Р. опре. на } \mathcal{A} \text{ т.к. не во всем опрег-но где бер } 2^{\mathbb{R}}$$

$$(\Omega, \mathcal{A}, \mathbb{P}) - \text{б.п.},$$

$\mathbb{P}$ - б-г-ат мере: неотр. число - вероят. Могут быть и отриц. из $\mathcal{A}$

$$\mathbb{P} : \mathcal{A} \rightarrow \mathbb{R} \text{ т.ч.}$$

- 1) $\forall A \in \mathcal{A} \mathbb{P}(A) \geq 0$
- 2) $\mathbb{P}(\Omega) = 1$
- 3) $\forall A_1, A_2, \dots \in \mathcal{A}, A_i \cap A_j = \emptyset, i \neq j$

$$\mathbb{P}(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mathbb{P}(A_n)$$

$$3) \forall A, B \in \mathcal{A}, A \cap B = \emptyset \Rightarrow \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$$

конечная система. 4) аксиома: $\forall A_1, \dots \in \mathcal{A}, \text{ т.ч. } A_i \subseteq A_j \Rightarrow$

$$\mathbb{P}(A_i) \leq \mathbb{P}(A_j) \quad 1 - \mathbb{P}(A) = \mathbb{P}(A^c) = \sum \mathbb{P}(A_i^c)$$

$\textcircled{2} P(\bigcup_{n=1}^{\infty} A_n) = \lim_{n \rightarrow \infty} P(A_n)$
 $\Rightarrow \dots \Rightarrow P(\bigcap_{n=1}^{\infty} A_n) = \lim_{n \rightarrow \infty} P(A_n) = 0/2$

$1) \Leftrightarrow 3) + 4)$
 $A \subseteq B \Rightarrow P(A) \leq P(B)$

Мера - бер 2), $(\cdot, \cdot)$ - м.б. с мерой

$\mathcal{L} = \mathbb{R}, \mathcal{A} = \mathcal{B}, \lambda((a, b)) = b - a$

Мера Лебесга - мерой-ле мере сумм на все действительные числа

$(\mathcal{B}_1, \mathcal{B}_2[a, b], \lambda)$ - б.н.

$\text{Def } (\mathcal{L}, \mathcal{A}), (\mathbb{R}, \mathcal{B})$ - м.к.р.-ла

м.б.м. мере $g: \mathcal{L} \rightarrow \mathbb{R}, \tau \in \mathcal{L}$

$g^{-1}(B) := \{\omega \in \mathcal{L}, g(\omega) \in B\} \in \mathcal{A} \forall B \in \mathcal{B} \quad (*)$

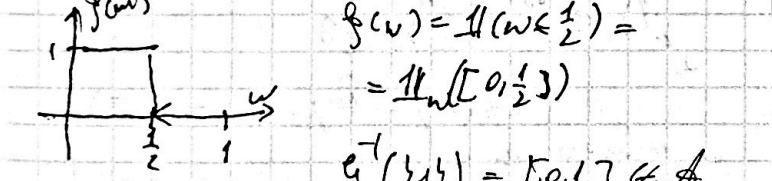
$\Rightarrow$ опре-ме $P_g(B) = P(g^{-1}(B)) = P(\{\omega: g(\omega) \in B\}) = P(g \in B), B \in \mathcal{B}$ - мер-е с.с. g

$(\mathbb{R}, \mathcal{B}, P_g)$ - б.н. - мер-е.

$(*) \Leftrightarrow (**): \forall x \in \mathbb{R} \{\omega: g(\omega) < x\} \in \mathcal{A}$
 $B = (-\infty, x)$

Пример мере. опре.

$\mathcal{L} = [0, 1], A = \{\omega \in \mathcal{L}\}$



$g(\omega) = 1(\omega \in \frac{1}{2}) = 1_{\omega \in [0, \frac{1}{2}]}$
 $g^{-1}(\{1\}) = [0, \frac{1}{2}] \notin \mathcal{A}$

$\Rightarrow P_g$ не опре-ме
 Или форм-е: 1) не гомоморфизм 2) не м.б.

$\mathcal{A}_1 = \{[0, \frac{1}{2}], \emptyset, \mathcal{L}, [\frac{1}{2}, 1]\}$

$g^{-1}(B) = \left\{ \begin{array}{lll} \mathcal{L}, & \text{if } B \in \mathcal{B}, & \text{if } B \in \mathcal{B} \\ [0, \frac{1}{2}] & \in & \notin \\ [0, \frac{1}{2}] & \notin & \notin \\ \emptyset & \notin & \notin \end{array} \right\} \in \mathcal{A}$

$\Rightarrow g$ - м.б. м.б. м.б.
 с-м $\mathcal{A}_2 \supseteq \mathcal{A}_1 \Rightarrow g$ - с.б. м.б. $(\mathcal{L}, \mathcal{A}_2)$

$\mathcal{A}_1 = \sigma(\mathcal{A})$ - м.б. м.б., м.б. м.б.
 с.б. g - м.б. ф.м. = $\sigma(\{g^{-1}(D), B \in \mathcal{B}\})$.

$(\mathcal{L}, \mathcal{A}_g, P)$ - м.б. м.б. g
 опре. мере: $g(\omega) = \text{const.}$

Мера-м.б. ф.м. = $\sigma(\{g^{-1}(D), B \in \mathcal{B}\})$.

$\text{Def } F_g = P_g((-\infty, x]) = P(\{\omega: g(\omega) \leq x\}) = P(g \leq x) = P(g < x) \quad x \in \mathbb{R}$

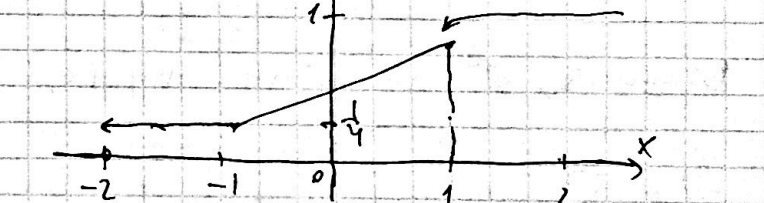
$\text{Def } F_g \Leftrightarrow P_g(B), B \in \mathcal{B}$

М.б. м.б. ф.м. = $\sigma(\{g^{-1}(D), B \in \mathcal{B}\})$.

- 1) $F(-\infty) = 0, F(+\infty) = 1$
- 2) F не уб. (вспомогательная лемма)
- 3) $F(x-0) = F(x) \quad \forall x \in \mathbb{R}$

Треб. $\forall F: \mathbb{R} \rightarrow \mathbb{R}$ угодн. 1)-3) $\exists$ б.н. $(\mathcal{L}, \mathcal{A}, P)$
 м.б. м.б. g м.б. $F_g(x) = P(g \leq x) \quad \forall x \in \mathbb{R}$.

Пример



$P(g = -2) = 0, P(-2 \leq g < 1) = \frac{3}{4}$

$P(g = -1) = 0, P(g > -2) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(g = 0) = 0, P(g \geq 2) = 1$

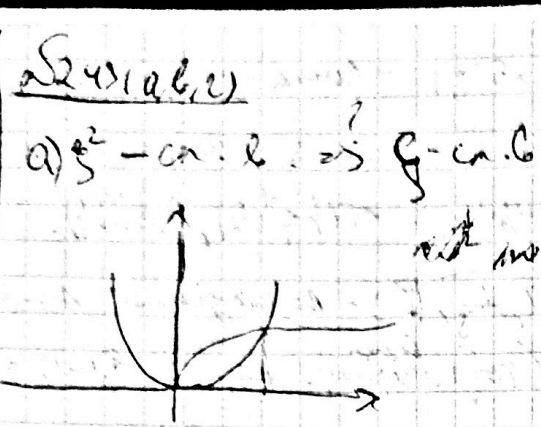
$P(g = 1) = \frac{1}{4}, P(g \geq 2) = 1$

$P(a \leq g \leq b) = P(g \leq b) - P(g < a) = F(b) - F(a)$

$P(g = a) = P(\bigcap_{n=1}^{\infty} [a \leq g < a + \frac{1}{n}]) = \lim_{n \rightarrow \infty} P(a \leq g < a + \frac{1}{n}) = F(a + \frac{1}{n}) - F(a) = F(a+0) - F(a)$

$F(0+0)$ - г.б. м.б.

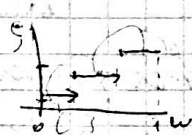
2.50
3.1 (2.3n.)
3.2



$\forall w \in B, g(w) \in B \cap A$
 $? \forall w \in B, g(w) \in B \cap A$

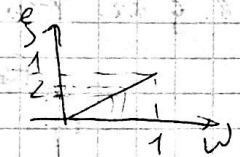
2.50

1) $g = \begin{cases} 1/4, & w \in [0, 1/4] \\ 1/2, & w \in (1/4, 3/4] \\ 1, & w \in (3/4, 1] \end{cases}$



$A = \{ \emptyset, [0, 1/3], [1/4, 1/3], [0, 1/4] \}$
 $B = \{ [1/4, 1/2], [0, 1/4] \cup [3/4, 1], \emptyset \}$

2) $g = \frac{w}{2}$



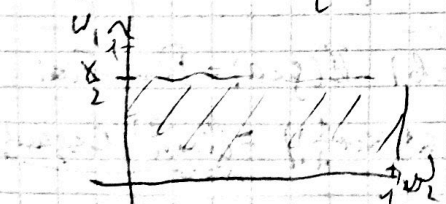
for $w \in [0, 1/2]$

b) $\emptyset, B$

3.1

3.2

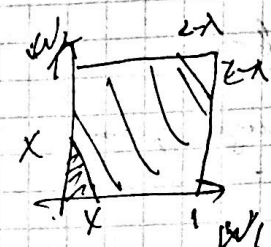
$\eta = w_1 + w_2, \eta = w_1 + w_2$



$P((g_1, g_2) \leq x) = P((w_1, w_2) \leq x)$
 $1 - \frac{x^2}{4}$
 $2w_1 \leq x$
 $w_1 \leq \frac{x}{2}$

2.4

$g = \begin{cases} 0, & (0, 1/2) \\ 1/4, & [1/2, 1] \end{cases}$ $A = \{ [0, 1/3], \emptyset \}$
 $\cos g \equiv 1 - c.b.$ $g^{-1}(0) = [0, 1/2] \notin A$
 $g^{-1}(1/4) = [1/2, 1] \in A$



$g = c.b. \Rightarrow g = c.b.$
 $A = [0, 1/3]$ $f(x) = \ln x - \ln 1/3$
 $f(g) = g - c.b.$

$f(1/2, A)$ $f(1/2, B)$ $f(1/2, B)$
 $f(1/2, B) \rightarrow f(1/2, B)$

$\ln - \ln$ $\ln x - \ln y$
 $\ln x - \ln y = \ln \frac{x}{y}$
 $\ln x - \ln y = \ln \frac{x}{y}$

$(\ln x) - \ln y = \ln \frac{x}{y}$
 $\ln x - \ln y = \ln \frac{x}{y}$

I. $\ln x - \ln y = \ln \frac{x}{y}$

$f(y) \in B$ $g(y) \in B$
 $g^{-1}(x) \in A$ $\forall x \in B$
 $(f \circ g)^{-1}(y) \in A$
 $g^{-1} \circ f^{-1}(y) = g^{-1}(y) \in A$
 $g^{-1}(f^{-1}(y)) \in A$

$$\xi = 1, w_1 = w_2; \xi = 0, w_1 \neq w_2, \eta = w_1, w_2$$

$$\xi + \eta = \begin{cases} w_1, w_2, w_1 \neq w_2 \\ 1 + w_1, w_2, w_1 = w_2 \end{cases} \xrightarrow{\text{ан.}} = w_1, w_2$$

$$w_2 \leq \frac{x}{w_1} \quad x \in [0, 1]$$

$$\int_0^1 \frac{x}{w_1} dw_1 + x = x \ln w_1 \Big|_x^1 + x = x(1 - \ln x) = F(x)$$

$$F(x) = \begin{cases} 0, & x \leq 0 \\ x(1 - \ln x), & 0 < x \leq 1 \\ 1, & x > 1 \end{cases}$$

$$a) \xi = \frac{w}{2}, \{x: \xi < x\} = \{x: \frac{w}{2} < x\} = \{x: w < 2x\} \quad x \in [0, 1]$$

$$\sigma(\xi) \ni \{0, 2x\}, 0 < x \leq \frac{1}{2}$$

$$b) \xi = \frac{1}{2} = \text{const}, \sigma(\xi) = \{\Omega, \emptyset\}$$

$$\xi^{-1}(B) = \Omega, B \ni \frac{1}{2}$$

$$\xi^{-1}(B) = \emptyset, B \not\ni \frac{1}{2}$$

$$F(x) = P(\xi < x) = P(\{w \in \Omega: \xi(w) < x\}), x \in \mathbb{R}$$

$$P_\xi(B) = P(\xi^{-1}(B)), B \in \mathcal{B}$$

$$(\mathcal{B}, \mathcal{A}, P) \xrightarrow{\xi} (\mathbb{R}, \mathcal{B}, P_\xi) - \text{измер. б.м.}$$

Дл. б.м. ξ (ее разг., г.р.) изм-е:

1) густотная, если $\exists B$ -те σ -алгебры, для которых т.з. $P(\xi \in B) = P_\xi(B) = 1$ (B-образные изм-е)

2) абс.-непрер., если $\exists p_\xi(y) \geq 0, y \in \mathbb{R}$:

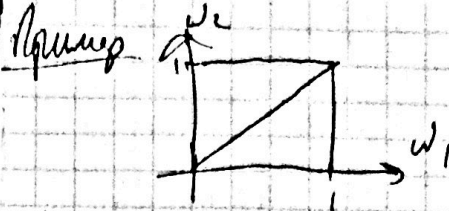
$$F_\xi(x) = \int_{-\infty}^x p_\xi(y) dy \quad \text{е) непрерыв. кр. изм-е}$$

3) суммируема, если $\exists$ б.м. $F_\xi(x)$ - непрерыв.

и изм-е т. разг. имеет меру 0 по разг.

$$\text{Дл. } x \in \mathbb{R} \rightarrow \text{т. разг. } \text{г.р. } F_\xi$$

$$\forall \varepsilon > 0 \quad F_\xi(x+\varepsilon) - F_\xi(x-\varepsilon) > 0$$



$$3) \text{---} \exists B \in \mathcal{B}: P(\xi \in B) = 1, \lambda(B) = 0.$$

$$B = [0, 1] \setminus \{\frac{1}{2}\} \in \mathcal{B}$$

$\mathcal{B}$ (Алгебра параметров)

$$\forall \text{ г.р. } F(x) = \sum_{i=1}^3 p_i F_{\alpha_i}(x) \geq 0, \sum_{i=1}^3 p_i = 1, \alpha_i \in \mathbb{R}$$

$$F(x) = p_1 F_{\alpha_1}(x) + p_2 F_{\alpha_2}(x) + p_3 F_{\alpha_3}(x) \quad \forall x$$

г.р.

$$a) \text{ густот., } B = \{x_1, x_2, \dots\}$$

$$F(x) = \sum_{i: x_i < x} P(\xi = x_i)$$

$$P(\xi = x) = \frac{\lambda^x}{x!} e^{-\lambda} = p_x, \quad x = 0, 1, 2, \dots$$

$$\{x_k, p_k\}_{k \in \mathbb{N}} - \text{разг. разг.-н. с.б.}$$

ξ	x_1	x_2	x_3	Σ
P	p_1	p_2	p_3	1

Дано: абс. и. с.б. ξ с разг. $p_\xi(x)$

$$F_\xi(x) = \int_{-\infty}^x p_\xi(y) dy$$

$$P(\xi < x) = P_\xi(x) = P(\xi \leq x) = \int_{-\infty}^x p_\xi(y) dy$$

$$p_\xi(x) = \frac{d}{dx} F_\xi(x)$$

$$P_\xi(x) = \int_{-\infty}^x p_\xi(y) dy$$

$$P_\xi(x) = \int_{-\infty}^x p_\xi(y) dy$$

$$P(\xi \in B) = \int_B p_\xi(x) dx$$

2.3
304
21.5.6
38

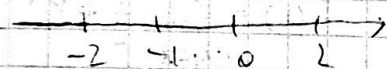
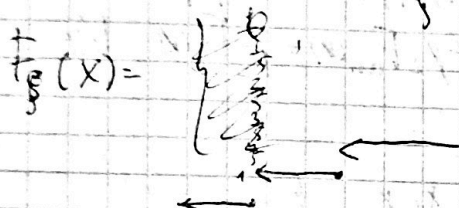
3.1

$$g, P(g=i) = \frac{1}{5}, i = -2, -1, 0, 2$$

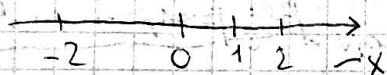
$$F_{\eta_1} = ?, F_{\eta_2} = ?, \eta_1 = -g, \eta_2 = |g|$$

$$F_{\eta_1} = P(\eta_1 < x) = P(-g < x) = P(g > -x) = 1 - P(g \leq -x) = 1 - F_g(-x)$$

$$F_{\eta_2}(x) = P(\eta_2 < x) = P(|g| < x) = P(-x < g < x) = F_g(x) - F_g(-x)$$



$$F_g(-x) = \sum_{k: X_k \leq -x} P(g = X_k) \Leftrightarrow$$



$$\Leftrightarrow \frac{1}{5} \sum_{k: X_k \leq x} 1 \Rightarrow F_{\eta_1} = 1 - \frac{1}{5} \sum_{k: X_k \leq -x} 1$$

Answer: a) $\eta_1: P(\eta_1=i) = \frac{1}{5}, i = -2, -1, 0, 2$

b) $P(\eta_2=0) = \frac{1}{5}, P(\eta_2=1) = \frac{2}{5}, P(\eta_2=2) = \frac{2}{5}$

3.2

$$P(g=k) = C/k(k+1), \sum_{k=1}^{\infty} \frac{C}{k(k+1)} = 1$$

$$S_n = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} = \frac{1}{n+1}$$

$$\lim_{n \rightarrow \infty} S_n = 1 - \frac{1}{n+1} \rightarrow 1 \Rightarrow C=1$$

$$a) P(g \geq 5) = 1 - P(g < 5)$$

$$b) P(\eta_1 \leq g \leq \eta_2)$$

3.3

$$p_g(x) = C/x^4, x \geq 1$$

$$p_g(x) = 0, \text{ for } x < 1$$

$$\int_1^{\infty} \frac{C}{x^4} dx = 1 = C \left[-\frac{1}{3x^3} \right]_1^{\infty} = \frac{C}{3} \left(\frac{1}{1^3} \right) = \frac{C}{3}$$

$$\Rightarrow \frac{C}{3} = 1 \Rightarrow C=3$$

$$d) \eta = 1/g \text{ r.v. from dep. } \Rightarrow$$

$$p_{\eta}(x) = \frac{1}{|g'(x)|} \cdot p_g(g^{-1}(x))$$

$$F_{\eta}(x) = P(\eta < x) = P\left(\frac{1}{g} < x\right) = P\left(\frac{1}{x} < g\right) = P(g > \frac{1}{x}) = 1 - F_g\left(\frac{1}{x}\right)$$

$$= 1 - \int_1^{\frac{1}{x}} \frac{3}{t^4} dt = 1 - \left[-\frac{1}{t^3} \right]_1^{\frac{1}{x}} = 1 - \left(-x^3 + 1 \right) = x^3$$

$$\Rightarrow p_{\eta}(x) = 3x^2$$

3.5

$$P(g \leq x) = 1 - e^{-\alpha x} (x \geq 0), p_g = \alpha e^{-\alpha x}$$

$$a) \eta_1 = \sqrt{g}, \delta_1 = \eta_2 = g^2$$

$$b) F_{\eta_1}(x) = P(\eta_1 < x) = P(g < x^2) =$$

$$= \int_0^{x^2} \alpha e^{-\alpha t} dt = 1 - e^{-\alpha x^2} = 1 - e^{-\alpha x^2}$$

$$\Rightarrow p_{\eta_1} = 2\alpha x e^{-\alpha x^2}$$

3.6

$$a) \eta_1 = \{g\}, \delta_1 \eta_2 = 1 - e^{-\alpha g}$$

$$F_{\eta_1}(x) = P(\{g\} < x) \Leftrightarrow$$

$$P(g \in [0, x)) = \int_0^x p_g(t) dt = \int_0^x \alpha e^{-\alpha t} dt = 1 - e^{-\alpha x}$$

$$p_{\eta_1}(x) = \frac{d}{dx} F_{\eta_1}(x) = \alpha e^{-\alpha x}$$

$$= \sum_{k=0}^{\infty} \alpha e^{-\alpha k} (1 - e^{-\alpha x}) = (1 - e^{-\alpha x}) \sum_{k=0}^{\infty} \alpha e^{-\alpha k}$$

$$= (1 - e^{-\alpha x}) \cdot 1 = 1 - e^{-\alpha x}$$

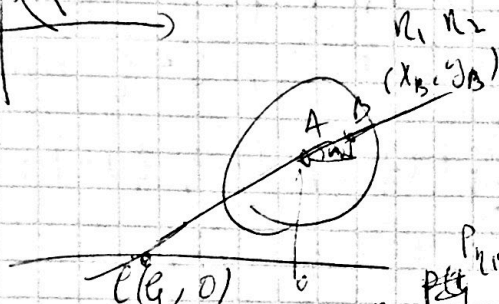
$$q_1 = 1, q = \frac{1}{e^x}; \quad \frac{1}{1 - \frac{1}{e^x}} = \frac{e^x}{e^x - 1}$$

$$\Rightarrow f_{q_1}(x) = (1 - e^{-x}) \cdot \frac{e^x}{e^x - 1} = \frac{1}{1 - e^{-x}} \cdot e^{-x} = \frac{1}{e^x - 1} \cdot e^{-x}$$

$$= \frac{1}{1 - e^{-x}}$$

3.8

g: $x^2 + (y-a)^2 = r^2, A = (0, a)$



$F(x) = ?$, $f(x) = ?$

$P_{n_2}(x, y) = \frac{1}{\pi a^2} \cdot \frac{1}{\pi a^2} = \frac{1}{\pi^2 a^4}$

$y = \frac{y_B - a}{x_B} x + a = 0$

$x = g = \frac{-a x_B}{y_B - a}$

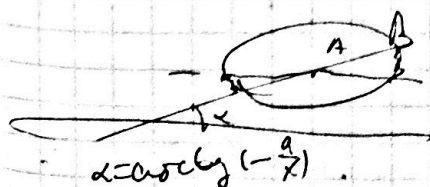
$F(y < x) = P\left(\frac{-a x_B}{y_B - a} < x\right) =$
 $= P(a(x - x_B) < x y_B) = P(a(x y_B - x_B) < x) =$
 $= P(x y_B < \frac{x - x_B}{a})$

$P(0 < x) = P\left(\frac{x - x_B}{y_B} < x\right) =$
 $= P(x(1 - y_B) < x_B) = P(x_B > x(1 - y_B))$

$P_{x, y_B} = P_x(u) \cdot P_{y_B}(v) =$

$F_{y_B}(x) = \int_{D_x}$

$D_x = \left(\frac{-a x_B}{y_B - a} < x\right)$



$P(y < x) = \frac{\arccos\left(\frac{-a}{x}\right)}{\pi}$

$= \frac{1}{2} + \frac{\arccos\left(\frac{x}{a}\right)}{\pi}$

$P(x) =$

Regulierung für unendliche X

$\int_{-\infty}^x \frac{1}{t} \mathbb{1}(t > 1) dt = \begin{cases} x < 1: 0 \\ x \geq 1: \int_1^x \frac{1}{t} dt = \frac{1}{x+1} \end{cases}$

F: $\begin{cases} \text{norm. auf} \\ x \rightarrow \infty \rightarrow 1 \\ x \rightarrow -\infty \rightarrow 0 \\ \text{regulierung.} \end{cases}$

$g: \begin{cases} \text{norm. auf} \\ x \rightarrow \infty \rightarrow 0 \\ x \rightarrow -\infty \rightarrow 1 \\ \text{regulierung.} \end{cases}$

$1 - P(g \leq \frac{1}{x}) = 1 - F_g(\frac{1}{x})$

$f_g(x) = 3x^2 \mathbb{1}(0 < x < 1)$

$f_g(x) = -\frac{F'_g(\frac{1}{x}) \cdot (\frac{1}{x})'}{f_g(\frac{1}{x})}$

$f_g(x) = (1 - e^{-x}) \mathbb{1}(x > 0)$

$F_g(x) = (1 - e^{-x}) \mathbb{1}(x > 0)$

$W(k \leq g < k+x) = W(g < k+x) - W(g < k)$

5.10

$g \sim K(a, b)$, $p(x) = \frac{1}{x} + \frac{b}{1 + b x^2}$

$g = \frac{1}{g}$

2 norm. kann:

$g = \frac{1}{g} = g$

$P(g < x) = P(g \geq x)$

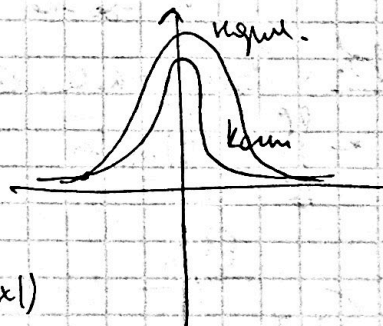
$F_g(x) = P\left(\frac{1}{g} < x\right) = P\left(\frac{1}{g} < x \mid g < 0\right) + P\left(\frac{1}{g} < x \mid g > 0\right)$

$\Rightarrow x > 0: F_g(x) = W(g < 0) + W(g > \frac{1}{x}) =$
 $= F_g(0) + 1 - F_g(\frac{1}{x})$

$\Rightarrow x < 0: W(\frac{1}{x} < g < 0) + 0 = F_g(0) - F_g(\frac{1}{x})$

3) $x = 0$ genau 2 norm. (mes. & 0 am anfang)

2) $P_g(x) = -P_g(\frac{1}{x}) \cdot (-\frac{1}{x^2}) \Rightarrow 1 = \frac{1}{x} + \frac{b}{1 + b x^2}$



$$\textcircled{2} \frac{1}{ax} \cdot \frac{1}{ax} = \frac{1}{a^2(x)} = p_3(x)$$

$$F_{p_3}(x) \stackrel{b.20}{=} P\left(\xi < \frac{x-a}{b}\right) = F_{\xi}\left(\frac{x-a}{b}\right)$$

$$p_{p_3}(x) = p_{\xi}\left(\frac{x-a}{b}\right) \cdot \frac{1}{b}, \quad b > 0$$

$$(Z, A, P) - \text{b.n.}; \quad \xi, \eta - \text{c.b.}$$

$$(\xi, \eta) - \text{u. l.u.s.p.} \quad \xi = (\xi_1, \dots, \xi_n), \eta = (\eta_1, \dots, \eta_n) - \text{c.b.}$$

$$\text{Def. } \xi, \eta - \text{u.s.p.} \Leftrightarrow P(\xi \in A_1, \dots, \eta \in A_n) = \prod_{i=1}^n P(\xi_i \in A_i, \eta_i \in A_i)$$

$$\xi_1, \dots, \xi_n - \text{u.s.p.} \Leftrightarrow \forall A_1, \dots, A_n \in \mathcal{B}$$

$$P(\xi_1 \in A_1, \dots, \xi_n \in A_n) = \prod_{i=1}^n P(\xi_i \in A_i)$$

$$\text{Th. } \xi_1, \dots, \xi_n - \text{u.s.p.} \Leftrightarrow$$

$$F_{\xi}(x_1, \dots, x_n) := P(\xi_1 < x_1, \dots, \xi_n < x_n) = \prod_{i=1}^n F_{\xi_i}(x_i)$$

$$\text{Def. } F_{\xi, \eta}(x, y) := P(\xi < x, \eta < y) \quad \forall x, y \in \mathbb{R}$$

$$x, y \rightarrow +\infty \Rightarrow 1 \quad \text{u.s.p.}$$

$$x, y \rightarrow -\infty \Rightarrow 0$$

$$\text{Th. } \xi, \eta - \text{u.s.p.} \Leftrightarrow$$

$$P(a \leq \xi \leq b, c \leq \eta \leq d) = F_{\xi, \eta}(b, d) - F_{\xi, \eta}(a, d) - F_{\xi, \eta}(b, c) + F_{\xi, \eta}(a, c)$$

$$\text{Def. } (\xi, \eta) - \text{a.u.} \Leftrightarrow \exists p(u, v) \geq 0, \text{ r.t.}$$

$$F(x, y) = \int_{-\infty}^x \int_{-\infty}^y p(u, v) du dv$$

$$\text{Th. } (\xi, \eta) - \text{a.u.} \Leftrightarrow P((\xi, \eta) \in B) = \int_B p(x, y) dx dy$$

$$= \int_B p(x, y) dx dy$$

$$\text{Def. } p$$

$$p_{\xi}(x) = \begin{cases} \frac{1}{b-a} & x \in (a, b) \\ 0 & x \notin (a, b) \end{cases} = \frac{1}{b-a} \mathbb{1}_{(a, b)}(x)$$

$$F_{\xi}(x) = P(\xi < x) = P(\xi < x-1) = P(\xi < \frac{x-1}{2}) = F_{\xi}\left(\frac{x-1}{2}\right)$$

$$\textcircled{2} p_{\eta} = \frac{1}{2} p_{\xi}\left(\frac{x-1}{2}\right) = \frac{1}{2(b-a)} \mathbb{1}_{(a, b)}\left(\frac{x-1}{2}\right)$$

$$\text{S. 1.4}$$

$$a) P(\xi_1 = -1) = \frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{1}{4}$$

$$P(\xi_2 = 1) = \frac{1}{24} + \frac{1}{6} + \frac{1}{8} = \frac{1}{4}$$

$$d) p_{\xi, \eta} = P(\eta_1 = i, \eta_2 = j)$$

$$\eta_1 = \xi_1 + \xi_2, \eta_2 = \xi_1 \xi_2$$

$$\text{S. 1.8}$$

$$p(\xi, \eta) = \frac{1}{\pi} \mathbb{1}_{(\xi, \eta) \in \text{hyperbola}}$$

$$P\left(|\xi| \leq \frac{3}{4}, |\eta| \leq \frac{3}{4}\right)$$

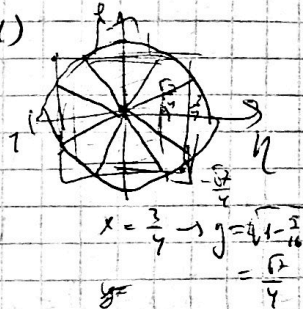
$$P\left(-\frac{3}{4} \leq \xi \leq \frac{3}{4}, -\frac{3}{4} \leq \eta \leq \frac{3}{4}\right) =$$

$$= \int_{-\frac{3}{4}}^{\frac{3}{4}} \int_{-\frac{3}{4}}^{\frac{3}{4}} p(u, v) du dv = \frac{1}{\pi} \int_{-\frac{3}{4}}^{\frac{3}{4}} \int_{-\frac{3}{4}}^{\frac{3}{4}} \mathbb{1}_{(u, v) \in \text{hyperbola}} du dv$$

$$= \frac{1}{\pi} \int_{-\frac{3}{4}}^{\frac{3}{4}} \int_{-\frac{3}{4}}^{\frac{3}{4}} \mathbb{1}_{(u, v) \in \text{hyperbola}} du dv =$$

$$= \frac{1}{\pi} \int_{-\frac{3}{4}}^{\frac{3}{4}} \int_{-\frac{3}{4}}^{\frac{3}{4}} \mathbb{1}_{(u, v) \in \text{hyperbola}} du dv =$$

$$P\left(\frac{1}{\pi} \int_{-\frac{3}{4}}^{\frac{3}{4}} \int_{-\frac{3}{4}}^{\frac{3}{4}} \mathbb{1}_{(u, v) \in \text{hyperbola}} du dv = 1\right)$$



$$\text{S. 1.9}$$

$$p_{\xi, \eta}(u, v) = \mathbb{1}_{(u, v) \in G}, \quad G = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1 - \frac{1}{2}u\}$$

$$p_{\xi}(x) = ?$$

$$p_{\eta}(y) = \int_{-\infty}^y \int_{-\infty}^{\infty} p_{\xi, \eta}(u, v) du dv$$

$$= \int_{-\infty}^y \int_{-\infty}^{\infty} p_{\xi, \eta}(u, v) du dv$$

$$\int_{-\infty}^y p_{\eta}(u, v) dv = \mathbb{1}_{(u, v) \in G} \cdot \int_{-\infty}^y dv = \mathbb{1}_{(u, v) \in G} (1 - \frac{1}{2}u)$$

$$\int_{-\infty}^x \mathbb{1}(u \in G_1) (1 - \frac{1}{2}u) du = \mathbb{1}(x \in G_1) \cdot (x - \frac{x^2}{2})$$

$$= \begin{cases} x, & x < 0 \\ 1, & x > 2 \end{cases}$$

3.20

$$f_{g_1, g_2}(u, v) = \begin{cases} C(u+v), & 0 \leq u \leq 1, 0 \leq v \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

a) $C = ?$ b) P_{g_1}, P_{g_2} c) $P_{\eta}, \eta = \max(g_1, g_2)$

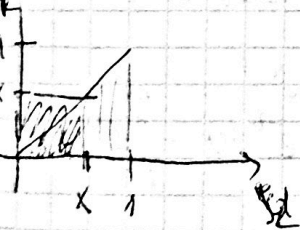
$$\int_0^1 \int_0^1 C(u+v) du dv = 1 \Rightarrow C \int_0^1 (uv + \frac{v^2}{2}) \Big|_{u=0}^{u=1} dv = 1$$

$$= C \int_0^1 (u + \frac{1}{2}) du = C \left(\frac{u^2}{2} + \frac{1}{2}u \right) \Big|_{u=0}^{u=1} = C \left(\frac{1}{2} + \frac{1}{2} \right) = C = 1$$

d) $P_{g_1} = \left(\int_0^1 C(u+v) dv \right) \Big|_{u=0}^{u=x} = x + \frac{1}{2} \quad (0 \leq x \leq 1)$

e) $P_{g_2} = \left(\int_0^1 C(u+v) du \right) \Big|_{v=0}^{v=x} = x + \frac{1}{2} \quad (0 \leq x \leq 1)$

f) $P(\eta < x) = P(g_1 < x, g_2 < x) + P(g_2 < x, g_2 \geq g_1) = \iint_{D_x} p_{g_1}(u) p_{g_2}(v) du dv$



$$D_x = \{(u, v) : \max(u, v) < x\}$$

$$F_{\eta}(x) = \iint_{D_x} (u + \frac{1}{2}v) du dv$$

$$D_x^* = D_x \cap \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1\}$$

Observe, also, that $x < 0 : P(\eta < x) = 0$
 $x > 1 : P(\eta < x) = 1$

For $0 \leq x \leq 1$:

$$\Rightarrow F_{\eta}(x) = \int_0^x \int_0^x (u + \frac{1}{2}v) du dv$$

$$\int_0^x (u + \frac{1}{2}x) du = \left(\frac{u^2}{2} + \frac{1}{2}xu \right) \Big|_{u=0}^{u=x} = \left(\frac{x^2}{2} + \frac{1}{2}x^2 \right) = \frac{x^2}{2}$$

3.11 $\Rightarrow P_{\eta}(x) = \begin{cases} 0, & x < 0 \end{cases}$

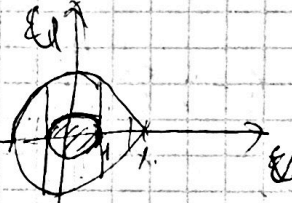
$$\left(\left(\frac{x^2}{2} + \frac{1}{2}x \right)^2 \right)' = 2 \left(\frac{x^2}{2} + \frac{1}{2}x \right) \left(x + \frac{1}{2} \right) = (x^2 + x) \left(x + \frac{1}{2} \right)$$

3.4

$$p_{g_1, g_2}(u, v) = \begin{cases} \frac{2}{\pi(u^2 + v^2)^3} & \text{for } u^2 + v^2 \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\eta = \sqrt{g_1^2 + g_2^2} = 1$$

$$P(\eta < x) = P(\sqrt{g_1^2 + g_2^2} < x) = F_{\eta}(x) \quad \begin{cases} x < 1: 0 \\ x > 1: 1 \end{cases}$$



$$\iint \frac{2}{\pi(u^2 + v^2)^3} du dv$$

$$u^2 + v^2 \leq x^2$$

$$\Rightarrow 4 \int_{-x}^x \int_{-\sqrt{x^2 - u^2}}^{\sqrt{x^2 - u^2}} \frac{2}{\pi(u^2 + v^2)^3} dv du$$

$$+ 4 \int_{-1}^0 \int_{\sqrt{1 - u^2}}^{\sqrt{x^2 - u^2}} \frac{2}{\pi(u^2 + v^2)^3} dv du$$

$$u^2 + v^2 = 1$$

$$u = \pm \sqrt{1 - v^2}$$

$$\Rightarrow 4 \int_{-1}^1 du \int_{-\sqrt{x^2 - u^2}}^{\sqrt{x^2 - u^2}} \frac{2}{\pi(u^2 + v^2)^3} dv$$

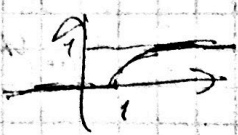
$$P(g_1^2 + g_2^2 < x^2) = \begin{cases} P(g_1^2 + g_2^2 < x^2) \\ 0, & x \leq 0 \end{cases}$$

$$= \iint_{u^2 + v^2 \leq x^2} p_{g_1, g_2}(u, v) du dv = \iint_{\substack{1 \leq u^2 + v^2 \leq x^2 \\ 0, x < 1}} \frac{2}{\pi(u^2 + v^2)^3} du dv =$$

$$= \int_0^{2\pi} \int_1^x \frac{2}{\pi^2} r dr d\theta = 4 \int_1^x \frac{r}{\pi^2} dr \Rightarrow P_{\eta}(x) = \frac{4}{\pi^2} x^2$$

$$P_{\eta}(x) = \frac{4}{\pi^2} \mathbb{1}(x \geq 1)$$

$$F_{\eta}(x) = (1 - \frac{1}{x^2}) \mathbb{1}(x \geq 1)$$



Формулы Чернова
 независимые случайные величины. н.в.

3) ξ, η - нез. с.в. с п.р. F_ξ, F_η и чп. p_ξ, p_η

$$1) F_{\xi+\eta}(x) = \int_{-\infty}^x \int_{-\infty}^x p_{\xi, \eta}(u, v) du dv = \int_{-\infty}^x \int_{-\infty}^x p_\xi(u) p_\eta(v) du dv =$$

$$= \int_{-\infty}^x p_\xi(u) F_\eta(x-u) du = \int_{-\infty}^x F_\xi(x-v) p_\eta(v) dv$$

$$2) p_{\xi+\eta}(x) = \int_{-\infty}^{\infty} p_\xi(x-y) p_\eta(y) dy = \int_{-\infty}^{\infty} p_\eta(y) p_\xi(x-y) dy$$

попеременная интегрирования

$$\xi - \eta = \xi + (-\eta), p_{-\eta}(x) = p_\eta(-x)$$

$$\Rightarrow \xi \pm \eta \Rightarrow \pm y \Rightarrow \int_{-\infty}^{\pm y} p_\xi(x+y) p_\eta(y) dy$$

ξ, η - н.з.р.с.в. $\sim \exp(\alpha)$; $\xi = \xi - \eta \sim \Gamma(1, \alpha)$
 (незав.)

$$p_{\xi \pm \eta}(x) = \int_{-\infty}^{\infty} p_\xi(x \mp y) p_\eta(y) dy$$

$p_\xi(x) = \lambda e^{-\lambda x} \mathbb{1}(x > 0)$, где $F_\xi(x)$ известна.

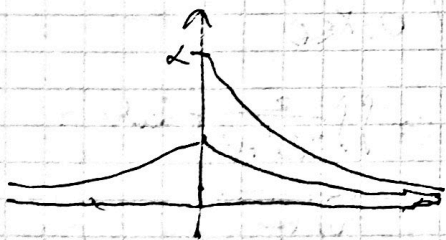
$$p_{\xi-\eta}(x) = \int_{-\infty}^{\infty} \lambda e^{-\lambda(x+y)} \mathbb{1}(x+y > 0) e^{-\mu y} dy$$

0v(-x) on max 1-min

$$\Leftrightarrow \int_{-\infty}^{\infty} \lambda e^{-\lambda(x+y)} \mathbb{1}(x+y > 0) e^{-\mu y} dy = \lambda \frac{e^{-\lambda x}}{-\lambda + \mu} \left[e^{-(\lambda+\mu)y} \right]_{y=0}^{\infty}$$

$$\Leftrightarrow \frac{\lambda}{\mu - \lambda} (1 + e^{-x(\mu - \lambda)}) = \begin{cases} \frac{\lambda}{\mu} e^{-\mu x}, & x > 0 \\ \frac{\lambda}{\lambda - \mu} e^{-\lambda x}, & x < 0 \end{cases}$$

$p_{\xi-\eta} = \frac{\lambda}{\mu} e^{-\mu |x|}$ - известность Лангесса.



неизвестно
 no calc
 calculation
 $|\xi - \eta| = \xi$

$\xi \sim \Gamma(1, \lambda), \lambda > 0$

известно, $F_\xi(\frac{1}{\lambda} \ln 2) = ?$, F

$$p_{\xi+\eta}(x) = ?, F_{\xi+\eta}(x) = P(\xi + \eta < x) =$$

известно

$$= \begin{cases} P(x < \xi < x) = F_\xi(x) - F_\xi(x-0), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$(p_\xi(x) + p_\xi(x-0)) \mathbb{1}(x > 0) = 2 p_\xi(x) \mathbb{1}(x > 0) =$$

$$= 2 \lambda e^{-\lambda x} \mathbb{1}(x > 0) \sim \exp(\lambda)$$

$\xi_1, \dots, \xi_n$ - нез. с.в.; $\xi_i \sim \exp(\alpha_i)$

известно. с.в. $\min_{1 \leq i \leq n} \xi_i$ и $\max_{1 \leq i \leq n} \xi_i$

$$1) p_{\min} i - \min_{1 \leq i \leq n} \xi_i - ? \quad 2) p_{\max} := \max_{1 \leq i \leq n} \xi_i = \frac{d}{dx} \left(\frac{\xi_1 + \dots + \xi_n}{n} \right)$$

$$F_{\min}(x) = P(\min_{1 \leq i \leq n} \xi_i < x) = 1 - P(\forall \xi_i \geq x) = 1 - \prod_{i=1}^n P(\xi_i \geq x) =$$

$$= 1 - \prod_{i=1}^n (1 - P(\xi_i < x)) = 1 - \prod_{i=1}^n (1 - F_{\xi_i}(x)) = 1 - \prod_{i=1}^n (1 - e^{-\alpha_i x}) =$$

$$= 1 - e^{-x \sum_{i=1}^n \alpha_i} \sim \exp(\sum_{i=1}^n \alpha_i)$$

§3.4(1)

ξ, η - о.н.р. $\sim \exp(1)$; $\lambda \xi \sim \exp(1)$

$$Y = \frac{\xi + \eta}{\xi - \eta}$$

$$P(Y < x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_\xi(u) p_\eta(v) du dv =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-u} e^{-v} du dv = \int_{-\infty}^{\infty} e^{-v} dv \int_{-\infty}^{\infty} e^{-u} du = \int_{-\infty}^{\infty} e^{-v} dv = 1$$

$0, x \leq 0$

ξ_1, ξ_2 - нез.; $\xi_i \sim \Gamma(\alpha_i, \beta_i) \subset \text{н.з.}$

$$p_{\xi_i}(x) = \frac{\lambda \beta_i^{\lambda}}{\Gamma(\lambda)} e^{-\lambda x} \mathbb{1}(x > 0), \quad \Gamma(\lambda) = \int_0^{\infty} t^{\lambda-1} e^{-t} dt$$

$$\xi_1 + \xi_2 \sim \Gamma(\lambda, \beta_1 + \beta_2)$$

$$\xi_1 + \dots + \xi_n \sim \Gamma(\lambda, \sum_{i=1}^n \beta_i)$$

$$\xi_1, \dots, \xi_n - \text{нез.} \sim \exp(\alpha) \Rightarrow \sum_{i=1}^n \xi_i \sim \Gamma(n, \alpha)$$

3.125?

$g_i \sim \exp(1)$ if $g_i \sim g_j$
 $F_g(x) = \begin{cases} 1 - e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

$F_{g_n}(x) = \begin{cases} P(g_n \leq x) \\ 0, x < 0 \end{cases}$

$P(g_1 \leq x) = \prod_{i=1}^n P(g_i \leq x) = \prod_{i=1}^n F_g(x) = (1 - e^{-x})^n$

$F_{g_n}(x) = P(g_1 + g_2 + \dots + g_n \leq x)$
 $g_1 = \frac{x}{n} \rightarrow g_n = \frac{x}{n}$

$= P(g_1 < x - (g_2 + \dots + g_n))$

$\Rightarrow P(g_1 + \dots + g_n < x) = (F_g(x/n))^n$

$P(g_1 + \dots + g_n \leq x) = \int_0^x \prod_{i=1}^{n-1} P(g_i \leq x - u) \cdot p_{g_n}(u) du$
 $= \int_0^x \lambda^n e^{-\lambda(x-u)} e^{-\lambda u} du = \lambda^n e^{-\lambda x} \int_0^x e^{-\lambda u} e^{\lambda u} du = \lambda^n e^{-\lambda x} \int_0^x 1 du = \lambda^n x^n e^{-\lambda x} / n!$

$\Rightarrow \int_{u_1 + \dots + u_n \leq x} p_{g_1}(u_1) \dots p_{g_n}(u_n) du_1 \dots du_n = \int_0^x du_1 \int_0^{x-u_1} \dots \int_0^{x-u_1-\dots-u_{n-1}} p_{g_1}(u_1) \dots p_{g_n}(u_n) du_2 \dots du_n$

3.34?

$\Gamma(x, \lambda) \approx p(x) = \frac{\lambda^x x^{x-1}}{\Gamma(x)} e^{-\lambda x}$
 $\int_0^1 (1-t)^{x-1} dt = \frac{\Gamma(x) \Gamma(1)}{\Gamma(x+1)}$

$P(g_1 \leq y) = \int_0^y \frac{\lambda^x x^{x-1}}{\Gamma(x)} e^{-\lambda x} dx$
 $x < 0, 0$

$\int_0^x (x-y)^{x-1} (y)^{y-1} dy$
 $\Rightarrow P(A) = \sum_i P(A|B_i) P(B_i)$

3.125
1744
276
3.38
3.328
3.26

3.126 ?

3.128

$P_g(x) = \int_0^x \int_0^{x-u} e^{-u_1} e^{-u_2} du_1 du_2$
 $\begin{cases} \frac{u_1}{u_1+u_2} < x \\ 0 < u_1, u_2 < 1 \end{cases}$

$u_1(1-x) < u_2 + u_1 x$

$\Rightarrow \int_0^x \int_0^{x-u} e^{-u_1} e^{-u_2} du_1 du_2$

3.322)

3.38

$P(g_1 < \frac{x}{1-x} \sum_{k=2}^n g_k) \approx \Gamma(1, n-1)$

$\int_0^x \int_0^{x-u} p_{g_1}(u) p_{g_2+\dots+g_n}(u) du$

$F_{g_1}(\frac{ux}{1-x})$

$P_g(x) = \int_0^x p_{g_1}(\frac{ux}{1-x}) \cdot (\frac{ux}{1-x})^{n-1} \cdot \frac{u^{n-1}}{(1-x)^{n-1}} e^{-u} du$
 $\cdot 1(u \geq 0) du = \int_0^x e^{-\frac{ux}{1-x}} \cdot 1(\frac{ux}{1-x} \geq 0) \cdot \frac{u^{n-1}}{(1-x)^{n-1}} du$

$\cdot e^{-u} du = \frac{1(\cos x \leq 1)}{(1-x)^{n-1}} \int_0^x u^{n-1} e^{-\frac{ux}{1-x}} du =$

$= (n-1)(1-x)^{n-1} 1(0 < x < 1)$

$g_1, g_2 \sim \exp(1) \Rightarrow \frac{g_1}{g_1+g_2} \sim U(0,1)$
 uniform (pobno-uznesen).

Onp $\square$ $g, \chi^2_n \sim \exp(1)$, $g \sim N(0,1), \chi^2_n \sim \chi^2_n$
 $\chi^2_n = g_1^2 + \dots + g_n^2, g_i \sim \exp(1)$

$\chi^2_n = \frac{g^2}{\chi^2_n/n} \sim St(n)$, $n=1 \Rightarrow \chi^2_1 = \frac{g^2}{1} = \frac{g^2}{1}$

$F_{\chi^2_n}(x) = P(g < x \sqrt{\chi^2_n/n}) = \int_0^x F_g(x \sqrt{\frac{y}{n}}) p_{\chi^2_n}(y) dy$

$$P_2 = \int_0^{\infty} f_3(x) \sqrt{\frac{2}{\pi}} \cdot \sqrt{\frac{2}{\pi}} \cdot P_{X_2}(y) dy =$$

$$= \int_0^{\infty} \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\sqrt{2\pi}} \cdot \frac{e^{-x^2/2}}{2} \cdot \left(\frac{1}{2}\right)^2 y^{2-1} \cdot e^{-y^2/2} dy \quad \ominus$$

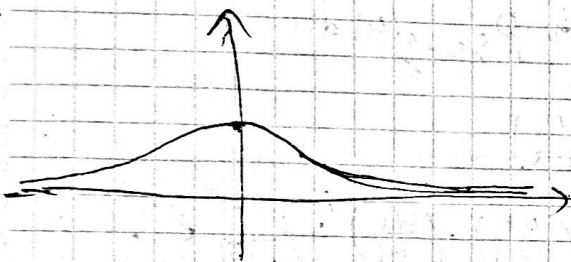
$$f(x) = \frac{x^{n-1}}{\Gamma(n)} e^{-x} \mathbb{I}(x>0)$$

$$\ominus \frac{2^{-n}}{\sqrt{2\pi}} \int_0^{\infty} y^{n-1} \exp(-y(-\frac{x}{2n} + \frac{1}{2})) dy \quad \ominus$$

$$y(-\frac{x}{2n} + \frac{1}{2}) = t, y = \frac{2t}{\frac{x}{n} + 1}$$

$$\ominus \dots \int_0^{\infty} \left(\frac{2}{\frac{x}{n} + 1}\right)^{\frac{n+1}{2}} \cdot f\left(\frac{n+1}{2}\right) =$$

$$= \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi} \Gamma(\frac{n}{2})} (1 + \frac{x^2}{n})^{-\frac{n+1}{2}} - \text{плотность распр. Стьюдента}$$



Для $n=1$: $\frac{\Gamma(1)}{\sqrt{\pi} \Gamma(\frac{1}{2})} (1+x^2)^{-1} = \frac{1}{\pi(1+x^2)}$

При $n \rightarrow \infty$: $\frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2) + O(\frac{1}{n})$
 "плотность Гаусса"

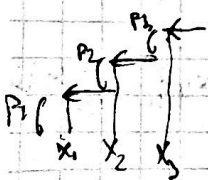
Формула Бириничи

Момент случайных величин

$\exists f$ -сл. на в.н. $(\Omega, \mathcal{F}, P)$ с F_f :

Опр $Ef = \int_{\Omega} f(\omega) P(d\omega) = \int_{-\infty}^{\infty} x dF_f(x)$

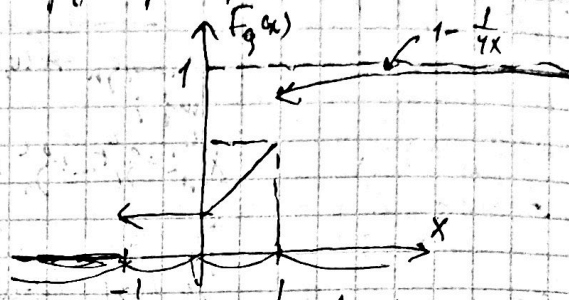
$\int_{-\infty}^{\infty} x f(x) dx$ — интеграл Лебег-Стилеса
 если f -а.м.
 $\sum x_i p_i$ — если f -групп. с
 если с.а.в.но — п.р. $\{x_i, p_i\}_{i \in \mathbb{N}}$
 тогда $\exists$.



Интеграл Римана: по разбиению

Лебега, Лебега-Стилеса: по образам значений

$\exists$ групп. распр. — e



$$Ef = 0 + (-1) \cdot \frac{1}{4} + 0 + \int_0^1 x d\left(\frac{1}{4}\right) + 1 \cdot \frac{1}{4} + \int_1^{\infty} x d\left(\frac{1}{4}\right)$$

$\Rightarrow$ не $\exists Ef$

Если $1 - \frac{1}{4x} \rightarrow 1 - \frac{1}{4x^2}$, то

$$Ef = \frac{5}{8}, \quad (Ef \text{ может не существовать})$$

Сб-ва E :

- 1) $Ec = c$
- 2) $E(cg) = cEf$
- 3) $E(f+g) = Ef + Eg$
- 4) $Eg(f) = \int g(x) dF_f(x)$
- 5) $f \leq g \Rightarrow Ef \leq Eg$
- 6) Если $f \geq 0 \Rightarrow \forall \varepsilon > 0 \quad P(f \geq \varepsilon) \leq \frac{Ef}{\varepsilon}$ (кр-ло Лебег-Стилеса)

$$Ef = Ef \mathbb{I}(f \geq \varepsilon) + Ef \mathbb{I}(f < \varepsilon) \geq \varepsilon P(f \geq \varepsilon) + 0 = \varepsilon P(f \geq \varepsilon)$$

И если y, f — второй кон. момент конечен
 $y := (y - Ef)^2$

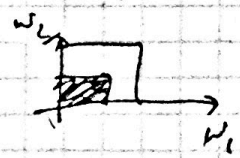
$$8) Ef(y) \geq y(Ef)^2 \quad y \geq 0 \quad (\text{кр-ло Чебышева})$$

$$9) |Ef| \leq (Ef^2)^{\frac{1}{2}} (Ef)^{\frac{1}{2}} : \frac{1}{p} + \frac{1}{q} = 1$$

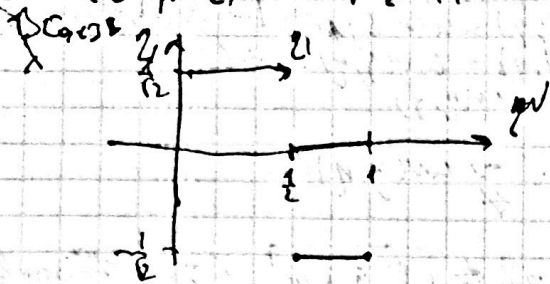
$$10) (Ef^p)^{\frac{1}{p}} \leq (Ef^q)^{\frac{1}{q}} \quad \forall 0 < p < q$$

$$11) (Ef+g)^p \leq (Ef)^p + (Eg)^p - \text{сильнее, чем 10}$$

$$12) f, g \text{ — к.м.} \Rightarrow Ef^2 = Ef \cdot Ef \mid Ef^2 = \int f^2 dP = \int f^2 dP$$



$$\Omega = \{(\omega_1, \omega_2) : 0 \leq \omega_1, \omega_2 \leq 1\}$$



3.82

$$\rho = \frac{\text{Cov}(X, Y)}{\sqrt{D(X)D(Y)}} = \frac{\text{Cov}(aY, bY)}{\sqrt{(a^2 D(Y))(b^2 D(Y))}} = \frac{ab \text{Cov}(Y, Y)}{ab \sqrt{D(Y)D(Y)}} = \text{Sign}(ab)$$

$$\rho(Y, Y) = \frac{\text{Cov}(Y, Y)}{\sqrt{D(Y)D(Y)}} = 1 > 0$$

$$\text{Cov}(Y, Y^2) = EY^3 - EY \cdot EY^2 = \frac{1}{4} - \frac{1}{2} \cdot \frac{1}{12} = \frac{1}{12}$$

$$EY^3 = \int x^3 dx = \frac{1}{4}; D(Y) = \frac{1}{12}, D(Y^2) =$$

$$= EY^4 - (EY^2)^2 = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$$

3.85

$$\int_{-\infty}^{\infty} x(1-e^{-x}) \cdot \frac{1}{2} e^{-x} dx = \int_0^{\infty} (1-e^{-x}) x e^{-x} dx$$

$$\int_0^{\infty} x(1-e^{-x}) \cdot \frac{1}{2} e^{-x} dx = \int_0^{\infty} (1-e^{-x}) x e^{-x} dx$$

3.86

n mean, cr. spaz: bur. karb. c mee. (aggr.)

$f_k = 1$ (k-e mehan nonano 6 chet karb.)

$$E \sum_{k=1}^n f_k \text{ (nagru at yru gi)} = \frac{1}{4} = 1$$

3.87

3.88

$$a) P(|Y - MY| < 3\sigma Y)$$

$$Y \sim N(\mu, \sigma^2) \Rightarrow MY = \mu \Rightarrow P(|Y - \mu| < 3\sigma)$$

$$D(Y) = \sigma^2 \quad P(-3\sigma + \mu < Y < 3\sigma + \mu)$$

$$\omega, \mu - \text{examp.} \Rightarrow P(-3 < Y < 3) \sim$$

$$= F(3) - F(-3) \approx 0.9987$$

$$\approx 0.9986 - 0.0014$$

$$2) EY = -\frac{1}{18} + \frac{1}{18} + 0 \cdot \frac{2}{9} = 0$$

$$DY = EY^2 - (EY)^2$$

$$= 0$$

$$\sum_{x=-1, 1, 0} x^2 \cdot P(Y=x) = \frac{2}{18} = \frac{1}{9} \Rightarrow P(|Y| < 3) = 1$$

3.89

$$P(Y_1, Y_2 = 0) = 1, P(Y_1 = 1) = P(Y_1 = -1) = \frac{1}{4}, i=1, 2$$

$$MY_1, MY_2, DY_1, DY_2, \text{Cov}(Y_1, Y_2)$$

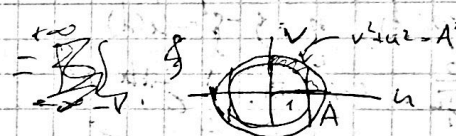
$$\text{Cov}(Y_1, Y_2) = EY_1 Y_2 - EY_1 EY_2$$

3.84

$$P_{B,C}(u, v) = \begin{cases} \frac{2}{\pi(u^2 + v^2)^2} & u^2 + v^2 \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E \sqrt{Y_1^2 + Y_2^2} = ?$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{u^2 + v^2} \cdot \frac{2}{\pi(u^2 + v^2)^2} du dv$$



$$v = \sqrt{1 - u^2}$$

$$\int_{-1}^1 \int_0^{\sqrt{1-u^2}} \frac{2}{\pi(u^2 + v^2)^2} du dv =$$

$$= 4 \cdot \frac{2}{\pi} \int_0^1 \int_0^{\sqrt{1-u^2}} \frac{1}{(u^2 + v^2)^2} du dv$$

$$u = r \cos \alpha \Rightarrow \int_0^{\frac{\pi}{2}} \int_0^1 \frac{1}{r^2} \cdot r dr d\alpha = \frac{\pi}{2} \cdot \left(\int_0^{\frac{\pi}{2}} r^2 dr \right)$$

$$u^2 + v^2 \geq 1 \Rightarrow r^2 = 1, r = 1$$

$$\Rightarrow \frac{8}{\pi} \left(\frac{1}{3} - \frac{1}{3 \cdot 4^3} \right) \Rightarrow$$

$$\frac{8}{3\pi}$$

$$\left(\frac{1}{3} \right) \Big|_0^1 = \frac{1}{3} - \frac{1}{3 \cdot 4^3}$$

3.104

$g, g_n \rightarrow u, g \sim \text{ReLU}$

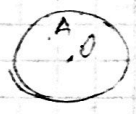
$$\begin{aligned} E\eta_n &= ? \quad \eta_n = \sum_{i=1}^n |g_{n,i} - g_i| \\ &= |g_1 - g_1| + |g_2 - g_2| + \dots + |g_n - g_n| \\ \Rightarrow E\eta_n &= E|g_1 - g_1| + \dots + E|g_n - g_n| \\ &= 0 \end{aligned}$$

$$\iint_{u,v} |u-v| \cdot \mathbb{1}_{(u,v) \in D} = \iint_{u,v} |u-v| \cdot \mathbb{1}_{(u,v) \in D} = \iint_{u,v} |u-v| \cdot \mathbb{1}_{(u,v) \in D}$$

$$\begin{aligned} u-v > 0, u > v \\ \textcircled{a} \int_0^1 \int_0^u (u-v) du dv &= \int_0^1 \left[\frac{u^2}{2} - uv \right]_{v=0}^u dv = \int_0^1 \left(\frac{u^2}{2} - u^2 \right) du = -\int_0^1 \frac{u^2}{2} du = -\frac{1}{6} \\ \textcircled{b} \int_0^1 \int_v^1 (v-u) du dv &= \int_0^1 \left[vu - \frac{u^2}{2} \right]_{u=v}^1 dv = \int_0^1 \left(v - \frac{v^2}{2} - v + \frac{v^2}{2} \right) dv = 0 \end{aligned}$$

$$\Rightarrow E\eta_n = \frac{n}{3}$$

3.86



$$\begin{aligned} E g &= \int_{\mathbb{R}} x p_g(x) dx = 2\pi \int_0^\infty x^3 e^{-x^2} dx \\ &= \frac{2\pi}{3} R^3 \end{aligned}$$

3.88

$$f(g, \eta) = \frac{\text{cov}(g, \eta)}{\sigma(g) \cdot \sigma(\eta)}, \quad D: \begin{matrix} 1 \\ \downarrow \\ 0 \end{matrix} \quad (g, \eta)$$

$$\begin{aligned} \text{cov}(g, \eta) &= E g \eta - E g \cdot E \eta \\ E g \eta &= \iint_{u,v} uv p_{g,\eta}(u,v) du dv = \iint_{u,v} uv \cdot \mathbb{1}_{(u,v) \in D} du dv \\ &= \int_0^1 \int_0^u uv du dv = \int_0^1 \left[\frac{uv^2}{2} \right]_{v=0}^u dv = \int_0^1 \frac{u^3}{2} du = \frac{1}{8} \end{aligned}$$

3.91

$$\begin{aligned} g, \eta \sim \text{unif}, E g &= E \eta = 0, E |g|^2 = 1, E |\eta|^2 = 1 \\ \Rightarrow E (g + \eta)^2 &= E g^2 + E \eta^2 \\ &= 1 + 1 = 2 \end{aligned}$$

3.107

$$\begin{aligned} E \eta_k &= a_k = \frac{1}{n-k+1}, \quad g \Rightarrow ? \\ E g &= (1-\eta_1)(1-\eta_2) \dots (1-\eta_{n-1}) \\ &= \prod_{i=1}^{n-1} E(1-\eta_i) = \prod_{i=1}^{n-1} \frac{n-i}{n-i+1} = \frac{1}{n} \end{aligned}$$

$$\begin{aligned} \ln X &= \sum_{i=1}^{n-1} (\ln(n-i) - \ln(n-i+1)) = \\ &= \ln(n-1) - \ln(n) + \ln(n-2) - \ln(n-1) + \dots \\ &= -\ln(n) \Rightarrow X = \frac{1}{n} \Rightarrow E g = \frac{1}{n} \end{aligned}$$

Kn. задача

$$\begin{aligned} \Gamma(n+1) &= n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n} \left(1 + \frac{1}{12n} + \dots\right) \\ \frac{\Gamma(n+k+1)}{\Gamma(n+1)} &\sim \left(\frac{n+k}{e}\right)^{n+k} \left(\frac{e}{n}\right)^n \sim \left(\frac{n+k}{n}\right)^n \left(\frac{n+k}{e}\right)^k \\ &\sim (1 + \frac{k}{n})^n \sim e^k \\ k \in \mathbb{N} \\ \text{Lem } k \in \mathbb{R}: \\ \frac{\Gamma(x+k)}{\Gamma(x)} &\sim x^k \quad \forall x, k \in \mathbb{R} \\ &\leq \frac{1}{x^{k-1}} \leq 1; \\ g &\sim \text{exp}(1), \quad E g^k = \int x^k e^{-x} dx = \Gamma(k+1) \end{aligned}$$

$$P(x+k) = E g^{x+k-1} = x^{k-k} + x+k-1 = k(x+1) + 1$$

$$P(x) = E g^{x-1} = 1$$

na avg. cen.

Uzgl: $E g^{xk} = (E g^x)^k = (E g)^k = 1^k = 1$

Rep-6 Stengene.

$$E(g|y) = (E(g))^y = 1^y = 1$$

$$\frac{1}{p} + \frac{1}{q} = 1$$

$\Rightarrow |g| = x^p = x^{\frac{1}{p}}$

$|g| = y^{\frac{1}{q}} = y^{\frac{1}{q}}$

$x, y \geq 0$

$E x^{\frac{1}{p}} y^{\frac{1}{q}} = (E x)^{\frac{1}{p}} (E y)^{\frac{1}{q}} = 1$

$0 \leq \delta \leq 1$

Gen. pasc-e §3.

$$P_{g|y}(x|y) = \frac{P_{g,y}(x,y)}{P_y(y)}$$

$$E g = E_y E(g|y), E(g|y) = \int x P_{g|y}(x|y) dx$$

$$\Rightarrow E(g|y) = y P_y(y) dy$$

$\Delta 3.191$ g, y - uog. $\sim \exp(\lambda)$

$$P_{g,y}(x,z) = \frac{P_{g,y}(x,z)}{P_y(z)} = \frac{\lambda^z e^{-\lambda z} \frac{1}{z} (0 < x < z)}{\lambda^z z e^{-\lambda z} 1(z > 0)}$$

$$F_{g,y}(x,z) = P(g < x, g+y < z) = \int_0^z P(g < x, g+y < z | y=y) \cdot P_y(y) dy$$

$$= P(g < x, g < z-y | y=y) = P(g < \min(x, z-y)) = 1 - e^{-\lambda(\min(x, z-y))}$$

$$\Rightarrow \int_0^z \lambda e^{-\lambda y} (1 - e^{-\lambda \min(x, z-y)}) dy \cdot 1(x \geq 0) =$$

dp

§3.189

$$1) P(g=y) = \sum_{k=1}^{\infty} P(g=y, y=k) \frac{P(y=k)}{P(g=k)} =$$

$$= p^2 \sum_{k=1}^{\infty} q^{2k-2} = \left(\frac{p}{q}\right)^2 \sum_{k=1}^{\infty} q^{2k} = \left(\frac{p}{q}\right)^2 \frac{1-q^2}{1-q^2} \cdot q^2 = p^2(1-q^2) = p^2 - p^2(1-p^2) = p^2 + p^4 = p^2(1+p^2)$$

$$6) P(g > y) = \sum_{k=1}^{\infty} P(g > y, y=k) P(y=k)$$

$$\frac{g > k}{= 1 - P(g \leq k)}$$

$$g = 1 - q$$

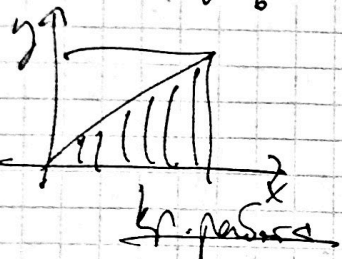
$$3) P E(g|g+y=l) = \dots$$

$$\Rightarrow \int_{-\infty}^{\infty} x P_{g|g+y=l}(x) dx = \frac{\int_{-\infty}^{\infty} x P_{g,y}(x,y) dx}{\int_{-\infty}^{\infty} P_{g,y}(x,y) dx}$$

$$= \frac{\sum_{k=1}^{\infty} k P(g=k, g+y=l)}{\sum_{k=1}^{\infty} P(g=k, g+y=l)} = \frac{P(g=l)}{P(g=l) + P(g=l-1)}$$

dp pasc-1

$$E|g_1 - g_2| = 2 \int_0^1 dx \int_0^x dy$$



$$D g_i = ? \quad D g_n = E \sum |g_{i+1} - g_i| + 2 \sum_{i < j} \text{cov}(g_i, g_j)$$

$$D|g_{i+1} - g_i| = E g_i^2 - (E g_i)^2$$

$$E g_i^2 = \int_{-\infty}^{\infty} x^2 p_g(x) dx = \int_0^1 x^2 \cdot 2(1-x) dx = \frac{1}{2}$$

$$D g_i = \frac{1}{2} - \left(\frac{1}{3}\right)^2 = \frac{1}{18}$$

$$\Rightarrow \frac{n}{18} + 2(n-1) \text{cov}(g_1, g_2)$$

$$\text{cov}(g_1, g_2) = E g_1 g_2 - E g_1 E g_2$$

commut - pasc. $g = \frac{1}{2} - g$

$$E g_1 g_2 = 2 E(g_2 - g_1)(g_2 - g_1) 1(g_2 > g_1) =$$

$$= 4 \int_0^1 \int_0^x (x_2 - x_1)(x_2 - x_1) dx_1 dx_2 dx_3$$

$$= 4 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{x_1+x_2} (x_1 - x_2)(x_1 - x_2) dx_3$$

$$= p q^{k-1} \cdot p q^{k-1} ; p \sum_{k=1}^{\infty} k p^{2k}$$

$$\sum_{k=1}^{\infty} k p^{2k} = |p-x| = \sum_{k=1}^{\infty} k x^{2k} = \frac{1}{x^2} \sum_{k=1}^{\infty} 2k x^{2k-1} =$$

$$= \frac{1}{2x} \left(\sum_{k=1}^{\infty} x^{2k} \right)' = \frac{1}{2x} \int (x^2 - x^4) dx =$$

$$= \frac{1}{2} \left(\frac{x^2}{2} - \frac{x^4}{4} \right) \Big|_{x=0}^1 = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

$$\Rightarrow E(\dots) = p^2 \cdot q^{2-1} \cdot \left(\frac{1}{4} - \frac{1}{8} \right)$$

3.150

a) $g > 0, P(g=k+1 | g > k) = p, k=0,1,\dots$

$$P(g=k+1) = \frac{P(g=k+1, g > k)}{P(g > k)}$$

$$P(g=k+1) = p \cdot \frac{P(g \leq k)}{P(g > k)} \quad \text{if } x_{k+1} = p(1 - \sum_{i=1}^k x_i)$$

$$P(g=1) = p(1 - P(g=1)) \Rightarrow P(g=1) = \frac{p}{1+p}$$

$$P(g=2) = p(1-p), P(g=3) = p(1-(p(1-p)))$$

$$= p(1-p+p^2) = p(1-p)^2$$

$$P(g=4) = p(1-(p(1-p+(1-p)^2))) = p(1-p)^3$$

$$P(g=k+1) = p(1 - \sum_{i=1}^k p(1-p)^{i-1})$$

$$\frac{p \sum_{i=1}^k (1-p)^{i-1}}{1-p} = \frac{p(1-p)(1-(1-p)^k)}{1-(1-p)^k}$$

$$\Rightarrow p(1 - \frac{p^2}{1-(1-p)^k}) = p(1-p)^k$$

$$1 - \frac{p^2}{1-(1-p)^k} = 1-p^k \Rightarrow 1-(1-p)^k - p^2 = (1-p)^k(1-(1-p)^k)$$

$$\Rightarrow 1-p^2 - 2(1-p)^k + (1-p)^{2k} = 0$$

$$p^2 = (1-(1-p)^k)^2$$

$$p = 1-(1-p)^k$$

$$1-p = (1-p)^k \quad \text{konst.}$$

3.151

$$P(g+q < z(x)) = ?$$

5) $g, q \sim R(0,1)$

$$P(g+q < z(x)) = \frac{P(g+q < z(x))}{P(g+q < 2)}$$

$$P(g+q < z) = \int_0^1 \underbrace{P(g+q < z)}_1 \frac{P_q(v)}{1} dv =$$

$$P(g < z-v) = F_g(z-v) = \begin{cases} 0, & z-v < 0 \\ z-v, & 0 \leq z-v \leq 1 \\ 1, & z-v > 1 \end{cases}$$

$$\Rightarrow \int_0^1 (z-v) \mathbb{1}(0 \leq z-v \leq 1) dv$$

$$\mathbb{1}(-1 \leq z-v \leq 0)$$

$$\mathbb{1}(-1+z \leq v \leq z)$$

$$\Rightarrow \int_{\max(0, -1+z)}^{\min(1, z)} (z-v) dv = \left(zv - \frac{v^2}{2} \right) \Big|_{\max(0, -1+z)}^{\min(1, z)}$$

3.152

$g_1, g_2 \sim \text{exp.}; g_1, g_1 + g_2 \sim ?$

$g_1 \sim \text{exp}(1) \Rightarrow R(0, g_1 + g_2)$

$g_1 \sim G(p) \Rightarrow R(0, g_1 + g_2), \text{ etc.}$

$$P(g_1 = k | g_2 + g_1 = s) = \frac{1}{s+1}, k=0,1,\dots$$

$g_1 \sim \text{Pois}(p) \Rightarrow B_1(g_1 + g_2, \frac{p}{1+p})$

$g_1 \sim B_1(n, p) \Rightarrow \text{unreg. exam.}$

$\forall x > 0, 0 < p \leq 1: \left(\frac{x}{x+p} \right)^{1-p} \leq \frac{P(x+p)}{x^p P(x)} \leq 1$

$g \sim \text{exp}(1) \Rightarrow P(g \geq xy | g \geq y) = P(g \geq x) \quad \forall x, y > 0$

$G(p)$ Distributions of random? (g. and. exp.)

$\text{Laguerre} \cdot N(\lambda) \sim \text{Pois}(\lambda), \lambda \sim \Gamma(r, t) \quad \forall r > 0$

$$\Rightarrow N \sim NB(z, p), p = \frac{t}{t+1}$$

$$\Leftrightarrow N, \lambda \sim \text{exp.}, N, \lambda \sim \text{Pois}(\lambda) \Rightarrow N, \lambda \stackrel{d}{=} N$$

$z \perp \lambda \sim N(0, \frac{1}{\lambda}), \lambda \sim \Gamma(\frac{2}{z}, \frac{1}{z}) \Rightarrow z \sim St(z)$

un-p. r. exp. p. mant.

$\Leftrightarrow g, \lambda \sim \text{exp.}, z \sim N(0, \eta)$

$z \perp \lambda \sim St(z)$

$g_1, g_2 \sim G(p), g_1 + g_2 \sim LG(2, p)$

$$P(\xi_1 + \xi_2 = k) = \sum_{i=0}^k P(\xi_1 = i) P(\xi_2 = k-i) =$$

$$= (k+1) p^k q^2, \quad k \neq \text{neg.} \text{ go 2 ymenaxa}$$

$$P(\xi_1 = k | \xi_1 + \xi_2 = 2) = \frac{1}{s+1}, \quad k = 0, 1.$$

$$P(\xi_1 \geq x+y | \xi_1 \geq y) = \frac{P(\xi_1 \geq x+y)}{P(\xi_1 \geq y)} = q^x = P(\xi_1 \geq x)$$

$$P(\xi_1 \geq y) = \sum_{i=y}^{\infty} p q^i = p q^y, \quad y \geq 0, \quad q < 1$$

the exp: $P(\xi_1 \geq y) = 1 - F_{\xi_1}(y) = e^{-\lambda y}, \quad y \geq 0.$

генер. ф. нел

$$\Rightarrow e^{-\lambda x} = P(\xi_1 \geq x)$$

$$E \xi_1^p \leq (E \xi_1^q)^{\frac{p}{q}} \cdot (E \xi_1^r)^{\frac{r}{r-p}} \quad \frac{1}{p} + \frac{1}{q} = 1, \quad p, q \geq 1$$

$$\frac{1}{p} = \delta, \quad \frac{1}{q} = 1 - \delta$$

$$E y^{\delta + 1 - \delta} \leq (E y)^{\delta} \cdot (E x)^{1 - \delta}$$

$$\Gamma(x+1) = \int_0^{\infty} y^{x+1-1} e^{-y} dy = E y^{x+1-1} = E y^{x+1-x-1} =$$

$$= E y^{x+1-1} = E y^{x+1-x-1} = E y^{x+1-x-1} \leq$$

$$\leq (E y^x)^{\delta} \cdot (E y^{x-1})^{1-\delta} = \Gamma(x+1)^{\delta} \cdot \Gamma(x)^{1-\delta} = x^{\delta} \Gamma(x)$$

$\frac{\Gamma(x+1)}{x^{\delta} \Gamma(x)}$ чане $\delta < 1$, јесен б рел.: $x = x + \delta$.

$\Rightarrow$ нисаме овећан.

$$N \sim N(2, p); \quad P(N=k) = \binom{p}{k+1} p^k q^k, \quad k=0, 1, \dots$$

$$P(N=k) = \int_0^1 P_{N,k}(x) \cdot P_{N,k}(x) dx \quad \text{e}$$

$$= \int_0^1 \frac{x^{2k} e^{-2x}}{\Gamma(2)} \cdot \frac{x^k e^{-x}}{k!} dx = \frac{1}{\Gamma(2) k!} \int_0^1 x^{2k+1} e^{-3x} dx$$

$$= \frac{\Gamma(2k+2) \Gamma(2)}{\Gamma(2) k! \Gamma(2)} \cdot \frac{1}{3^{2k+2}} \cdot \frac{1}{2k+1} \cdot \frac{1}{2k+2}$$

C

kn. pabeta

$$(R, \mathcal{A}, P) - \text{b.n.} \quad | E f = \int_{\mathbb{R}} f(x) P(dx) = \int_{\mathbb{R}} x dF_{\xi}(x)$$

$$g: \Omega \rightarrow \mathbb{R} \quad \text{r.} \quad K \in \mathcal{B} \quad g^{-1}(K) \in \mathcal{A}$$

$$\text{Onp } f_{\xi}(t) = E e^{it\xi} = E \cos(t\xi) + i E \sin(t\xi) \in \mathbb{C}$$

$$\text{e} \quad \begin{cases} \int_{-\infty}^{\infty} e^{itx} p_{\xi}(x) dx, & \text{f-a.n.} \\ \sum_k e^{itx_k} P(\xi = x_k), & \text{f-групп. } \{x_1, x_2, \dots\} \end{cases}$$

Ch-bal

- 1) $|f(t)| \leq 1, \quad f(0) = 1$
- 2) еам ξ -непр. генер. ч. б. $\Rightarrow f_{\xi}(t) = \varphi_{\xi}(e^{it})$
- 3) $f_{\xi}(t) \leftrightarrow F_{\xi}, \quad F_{\xi}(x) = \frac{1}{2} + \frac{i}{2\pi} \text{v.p.} \int_{-\infty}^{\infty} \frac{e^{itx}}{-it} f_{\xi}(t) dt$
- 4) ξ, η -непр. $\Rightarrow f_{\xi+\eta}(t) = f_{\xi}(t) f_{\eta}(t) \quad \forall x \in \mathbb{R}$
- 5) $f_{a\xi+b}(t) = e^{itb} f_{\xi}(at)$

$$6) E \xi^k = i^{-k} f^{(k)}(0),$$

еам $E |\xi|^n < \infty \Rightarrow \exists f^{(n)}(t) \quad \forall t \in \mathbb{R}$

$$|f^{(n)}(t)| \leq E |\xi|^n$$

$$b.2) f(t) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} t^k + o(t^n) = \sum_{k=0}^{\infty} \frac{(ik)^k E \xi^k}{k!} + o(t^n)$$

Монг. ф. он-се јул генер. ч. б. беман.

the gen-ba 3): $\int_0^{\infty} \frac{\sin tx}{x} dx = \frac{\pi}{2}$

Еам $f \in L_1(\mathbb{R})$ ($\int |f(t)| dt < \infty$), то

- (i) F_{ξ} -a.n. ($\exists p_{\xi}$ -m.)
- (ii) $p_{\xi}(x)$ -непр. ч он.
- (iii) $p_{\xi}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} f_{\xi}(t) dt$

3) Еам $f_{\xi}(t)$ павн. непр. на $\mathbb{R}$.

4) Еам $\beta_n := E |\xi|^n < \infty$, то $f^{(n)}(t) = E |\xi|^n e^{it\xi}$

- (i) $\exists f^{(n)}(t) \quad \forall t \quad |f^{(n)}(t)| \leq \beta_n$
- (ii) $E \xi^n = i^{-n} f^{(n)}(0) \quad \forall n \in \mathbb{N}$
- (iii) $f(t) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} t^k + o(t^n) = \sum_{k=0}^{\infty} \frac{E \xi^k (ik)^k}{k!} + o(t^n), \quad t \rightarrow 0$

Осрпел неврно тронато еам n -дета $4.95, 4.96$ генер. ч.

5) $f_{-\xi}(t) = f_{\xi}(-t) = \overline{f_{\xi}(t)}$

6) $g \stackrel{d}{=} -g \Leftrightarrow f_g(t) \in \mathbb{R} \Leftrightarrow f_g(t) = f_g(-t)$

Задача 1

$f \sim \Gamma(r, \alpha), p(x) = \frac{\alpha^r x^{r-1}}{\Gamma(r)} e^{-\alpha x} \mathbb{1}(x > 0)$

$f(t) = \mathbb{E} e^{itx} = \int_{-\infty}^{\infty} e^{itx} \cdot p(x) dx = \frac{\alpha^r}{\Gamma(r)} \int_0^{\infty} e^{itx} x^{r-1} e^{-\alpha x} dx$

$\cdot x^{r-1} e^{-\alpha x} \mathbb{1}(x > 0) dx, r=1$

② $f(t) = \alpha \int_0^{\infty} e^{(it-\alpha)x} dx = \frac{\alpha}{it-\alpha} (e^{(it-\alpha)x}) \Big|_0^{\infty} =$

$= 0 - \frac{\alpha}{it-\alpha} = \frac{\alpha}{\alpha-it}$

$f_1, f_2, \dots, f_n$ н.о.р.-с.б. $\sim \text{Pois}(\lambda)$

2) $f_1 + f_2 \sim \Gamma(2, \alpha) \Rightarrow f_2(t) = \left(\frac{\alpha}{\alpha-it}\right)^2, \forall t \in \mathbb{R}$

Примеры генераторов:

Определены f -коэф. генераторов с.б.

$\Psi_f(z) = \mathbb{E} z^f, |z| \leq 1, z \in \mathbb{C}$
 $= \sum_{k=0}^{\infty} z^k P(f=k)$

1) $|\Psi_f(z)| \leq 1 = \Psi_f(1)$

2) $f_g(t) = \Psi_g(e^{it}), \forall t \in \mathbb{R}$

3) $\Psi_f \leftrightarrow F_f$

4) f, g -коэф. $\Rightarrow \Psi_{f+g}(z) = \Psi_f(z) \cdot \Psi_g(z)$

5) $\Psi_{f^{(n)}}(z) = \mathbb{E} f(f-1) \dots (f-n+1) z^f = \mathbb{E} f^{(n)} z^f$

Задача: найти генератор $\Psi(z) = \frac{1}{4}(z+1)^2$

$\Psi(z) = e^{\lambda(z-1)} = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k z^k}{k!} \sim \text{Pois}(\lambda)$

Задача 5

$V \sim \text{Pois}(\lambda), \mu_n \sim \text{Dir}(n, p)$ - коэф.

$\Psi_{\mu}(z) = \mathbb{E} z^{\mu} = \mathbb{E} g(\mu) = \mathbb{E} g(\mu) \cdot \sum_{n=0}^{\infty} \mathbb{1}(V=n)$

② $\sum_n g(\mu) \mathbb{1}(V=n) = \sum_n \mathbb{E} g(\mu) \mathbb{E} \mathbb{1}(V=n)$

$g(\mu) \cdot \mathbb{1}(V=n) \quad \mathbb{E} z^{\mu} = \Psi_{\mu}(z)$

$\rightarrow$

$\mathbb{E} z^{\mu} = \sum_{n=0}^{\infty} P(V=n) \Psi_{\mu}(z)^n \exp\{\lambda p(z-1)\} \sim \text{Pois}(\lambda p)$

Задача 6

Задача 6.5

1) $\Psi_{\mu}(z) = e^{\lambda p(z-1)}$

$f_{\mu}(t) = e^{\lambda p(e^{it}-1)}$

$\mathbb{E} \mu = -i f'(0) = -i e^{\lambda p(e^{it}-1)} \cdot \lambda p e^{it} \Big|_{t=0} =$

$\ominus \lambda p$

$\mathbb{E} \mu = \lambda p$

Задача 6.6

а) $\Psi(z) = M z^f; \mathbb{E} f = (\Psi(e^{it}))'(0)$

$\mathbb{D} f = \mathbb{E} f^2 - (\mathbb{E} f)^2$

$= \mathbb{E} f^2 - (\mathbb{E} f)^2$

$\mathbb{E} (f - \mathbb{E} f)^2 = \mathbb{E} (f^2 - 2f \mathbb{E} f + (\mathbb{E} f)^2) = \mathbb{E} f^2 - 2\mathbb{E} f \mathbb{E} f + (\mathbb{E} f)^2 = (\mathbb{E} f)^2$

$\ominus \mathbb{E} f^2 - 2\mathbb{E} f \mathbb{E} f + (\mathbb{E} f)^2 = (\mathbb{E} f)^2$

б) $f_1 = f/2, f_2 = 2f, f_3 = -f, f_4 = f/2 - 2f = -3/2 f$

f_1, f_2 - коэф. н.о.р. f_3, f_4 - коэф. н.о.р.

$\Psi_{f_1}(z) = \mathbb{E} z^{f_1} = \mathbb{E} z^{f/2}$

б) $a > 0, \mathbb{E} \frac{1}{f+a} = \int_0^1 z^{a-1} \Psi(z) dz$

$\sum_{k=0}^{\infty} P(f=k) z^k \Rightarrow \mathbb{E} f$

Задача 7.5

$\Psi_f = \mathbb{E} z^f = \mathbb{E} z^{\sum_{n=0}^{\infty} \mathbb{1}(V=n)} = \sum_{n=0}^{\infty} \mathbb{E} z^{\sum_{n=0}^{\infty} \mathbb{1}(V=n)} \mathbb{1}(V=n)$

$= \sum_{n=0}^{\infty} \mathbb{E} z^{\sum_{n=0}^{\infty} \mathbb{1}(V=n)} \mathbb{1}(V=n) = \sum_{n=0}^{\infty} \mathbb{E} z^{\sum_{n=0}^{\infty} \mathbb{1}(V=n)} \cdot \frac{P(V=n)}{P(V=n)}$

Задача 7.8

Сумма $\mathbb{E} e^{itx} = \mathbb{E} e^{itx} = \mathbb{E} e^{itx}$

$f_g(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} f_g(t) dt$

Решение

Контроль

$g_1, g_2, \dots$ - с.б.

$$1) g_n \xrightarrow{n.u.} g \Leftrightarrow P(\{g_n(\omega) \rightarrow g(\omega)\}) \xrightarrow{n \rightarrow \infty} 1$$

$$2) g_n \xrightarrow{P} g \Leftrightarrow \forall \varepsilon > 0 \quad P(|g_n - g| > \varepsilon) \rightarrow 0$$

$$3) g_n \xrightarrow{L^2} g \Leftrightarrow E|g_n - g|^2 \rightarrow 0$$

$$4) g_n \xrightarrow{d} g \Leftrightarrow F_{g_n}(x) \rightarrow F_g(x) \quad \forall x, \text{ где } F_g(x) - \text{н.к.}$$

$$\Leftrightarrow E g(g_n) = \int g(x) dF_{g_n}(x) \rightarrow \int g(x) dF_g(x) \quad \forall \text{ г.к.}$$

$$5) [Th] (н.к.) \{f_n(b)\}_{n \geq 1} - \text{х.г.}$$

$$\text{Если } f_n(t) \rightarrow f(t) \quad \forall t \in \mathbb{R} \text{ и } f(t) \text{ н.к.}$$

$$f(t) = 0 \quad \forall t \in \mathbb{R}$$

$$(i) f(t) = E e^{itg} \quad \text{где н.к. с.б. } g$$

$$(ii) g_n \xrightarrow{d} g$$

$$g_n \xrightarrow{d} g \Leftrightarrow g_n \xrightarrow{P} g \Leftrightarrow g_n \xrightarrow{L^2} g \Leftrightarrow f_{g_n}(t) \rightarrow f_g(t) \quad \forall t$$

$$Задача. Справедливо доказать; имеем$$

$$E g_i \rightarrow E g$$

$$g_n \xrightarrow{d} c \Rightarrow g_n \xrightarrow{P} c$$

$$P(g_n < x) \rightarrow 1(c < x) \quad \forall x \neq c \quad \Rightarrow P(|g_n - c| > \varepsilon) \rightarrow 0$$

$$A \supset B, P(A) \leq P(B)$$

$$P(g_n < x) = P(g_n < x, |g_n - c| > \varepsilon) +$$

$$+ P(g_n < x, |g_n - c| \leq \varepsilon)$$

$$P(|g_n - c| \leq \varepsilon) = P(|g_n - c| \leq \varepsilon, g_n < x) +$$

$$+ P(|g_n - c| \leq \varepsilon, g_n \geq x)$$

$$\geq P(|g_n - c| \leq \varepsilon, x < c - \varepsilon) + P(|g_n - c| \leq \varepsilon, x \geq c + \varepsilon)$$

$$= P(|g_n - c| \leq \varepsilon, |x - c| \geq \varepsilon)$$

$$2) \text{ а) 4.97}$$

$$\cos^2 t = 1 - \frac{t^4}{2!} + \dots = 1 - \frac{t^4}{2} + o(t^4) \quad t \rightarrow 0$$

$$-\frac{1}{2} = \frac{f^{(4)}(0)}{4!} \quad ? \quad f^{(4)}(0) = -12$$

$$E g^k = (-i)^k f^{(k)}(0) \quad \text{н.к. } k\text{-е число}$$

$$E g^2 = 0; E g^4 < 0 \Rightarrow I$$

$$b) \cos^2 t - \text{аб.к. пар.}$$

$$\Rightarrow \cos^2 t \text{ пар. г.к. пар.}$$

$$\cos^2 t = \frac{e^{it} + e^{-it}}{2}$$

$$P(g=1) = 1, P(g=-1) = 1$$

$$P(g=1/2) = 1/2, P(g=-1/2) = 1/2$$

$$2) \cos^2 t \xrightarrow{d} 1$$

$$t_1 = 0, t_2 = (2\pi)^{1/2} \Rightarrow 2t_2 \Rightarrow f(\dots) \neq 1$$

$$3) \cos^2 t \xrightarrow{L^2} 1$$

$$t = \frac{\pi}{2} \Rightarrow \text{возмож. г.к.}$$

$$1 + \frac{2}{3} \left(-\frac{t^2}{2} + \frac{t^4}{4!} + o(t^4) \right) + \frac{2}{3} \left(-\frac{t^2}{2} + \frac{t^4}{4!} + o(t^4) \right)$$

$$E g^2 = 1; E g^4 = 0 \Rightarrow g = 0$$

$$\text{но пар. г.к. } \neq 0$$

$$g) \text{ н.к. по вер. } \Rightarrow \text{не пар.}$$

$$3) E g^2 = 1; E g^4 = 0 \Rightarrow g = 0$$

$$\text{но пар. г.к. } \neq 0$$

$$1) \text{ н.к. по вер. } \Rightarrow \text{не пар.}$$

$$2) \text{ н.к. по вер. } \Rightarrow \text{не пар.}$$

$$3) \text{ н.к. по вер. } \Rightarrow \text{не пар.}$$

$$f(t) = (1-t)^2$$

$$g = g_1 + \dots + g_n - \text{с.б.}$$

$$Dg = \sigma^2 E V + a^2 P V$$

Задача $f(t) = E e^{itg}$, $g(t) = E z^t$

$$\Rightarrow E e^{itg} = g(f(t)) \quad E z^t = g(\varphi(t))$$

Вспомог. задача: $\eta_n \sim B(n, p)$, $V = \text{Pois}(1)$

$$\eta_n = \sum_{i=1}^n \xi_i, \quad \xi_i = \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$$

$$f(t) = e^{\lambda(t-1)}$$

$$p(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-itx} f(t) dt$$

$$\Rightarrow \frac{1}{2\pi} \left(\int_{-1}^0 e^{-itx} (1+t) dt + \int_0^1 (1-t) e^{-itx} dt \right)$$

$$1) \int_0^1 e^{-itx} t dt = -\frac{1}{ix} \int_0^1 t d e^{-itx} = -\frac{1}{ix} \left(t e^{-itx} \Big|_0^1 - \int_0^1 e^{-itx} dt \right)$$

$$2) \int_{-1}^0 e^{-itx} dt = -\frac{1}{ix} \int_{-1}^0 d e^{-itx} = -\frac{1}{ix} e^{-itx} \Big|_{-1}^0$$

$$3) \int_{-1}^0 e^{-itx} dt = -\frac{1}{ix} (e^{-ix} - 1)$$

$$4) \int_0^1 t e^{-itx} dt = -\frac{1}{ix} \left(e^{-ix} + \frac{1}{ix} (e^{-ix} - 1) \right)$$

$$\Rightarrow 1) + 2) + 3) - 4) = -\frac{1}{ix} e^{-ix} + \frac{1}{x^2} (1 - e^{-ix}) - \frac{1}{ix} (1 + e^{-ix})$$

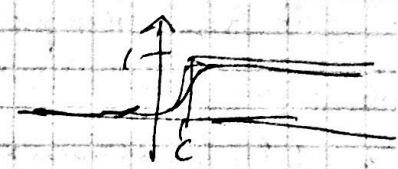
Кн. парадокс

$$g_n \xrightarrow{d} C \Rightarrow g_n \xrightarrow{P} C$$

$$P(g_n < x) \rightarrow \begin{cases} 0, & x \leq c \\ 1, & x > c \end{cases}$$

$$P(|g_n - c| < \varepsilon) \rightarrow 1$$

$$P(c - \varepsilon < g_n < c + \varepsilon) \geq P(c - \varepsilon_2 \leq g_n \leq c + \varepsilon) = F_{g_n}(c + \varepsilon) - F_{g_n}(c - \varepsilon_2) = 1$$



Закон больших чисел

Пусть $\xi_1, \xi_2, \dots$ — с.б. на $(\Omega, \mathcal{F}, P)$

$$\xi_i = a_i, \quad i = 1, 2, \dots$$

Доп. 1. Если $\{\xi_n\}$ с.б. ξ , то

$$\frac{\sum_{i=1}^n \xi_i}{n} \xrightarrow{P} 0$$

Лем. 354 в форме Чебышева

Пусть $\xi_1, \xi_2, \dots$ — н.о.р.с.б. $E \xi_i = a < \infty$

$$E \xi_i = a, \quad D \xi_i = \sigma^2 < \infty$$

$$P\left(\left|\frac{\sum_{i=1}^n \xi_i}{n} - a\right| > \varepsilon\right) \leq \frac{D\left(\frac{\sum_{i=1}^n \xi_i}{n}\right)}{\varepsilon^2} = \frac{\sum_{i=1}^n D \xi_i}{n^2 \varepsilon^2} = \frac{n \sigma^2}{n^2 \varepsilon^2} = \frac{\sigma^2}{\varepsilon^2 n} \rightarrow 0$$

Лем. 354 в форме Хинчина

Пусть $\xi_1, \xi_2, \dots$ — н.о.р.с.б. $\{E \xi_i = a < \infty\}$

$\Rightarrow$ где н.о.р.с.б. 354

X -с.б., $EX < \infty$, то $\varphi(t) = 1 + itEX + o(t)$

$$\text{Пусть } S_n = \sum_{i=1}^n \xi_i$$

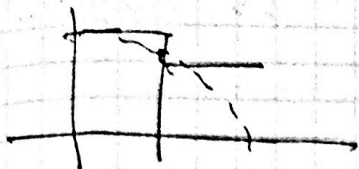
$$\varphi_{S_n}(t) = E \exp \left\{ it \sum_{i=1}^n \xi_i \right\} = \prod_{k=1}^n E \exp \left\{ it \xi_k \right\} = \prod_{k=1}^n \varphi_{\xi_k} \left(\frac{t}{n} \right)$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{itEX}{n} + o\left(\frac{t}{n}\right) \right)^n = e^{itEX} = \varphi_{EX}(t)$$

$$\varphi_n(t) \rightarrow \varphi(t) \Leftrightarrow \xrightarrow{d} C \rightarrow$$

$$\Rightarrow \sum_{i=1}^n \xi_i \xrightarrow{P} a$$

$1 < \frac{\pi}{2}$



$\text{с.в. } \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
 $(1 - \frac{t^2}{4m} + o(t^2))^{-\frac{m}{2}} = e^{-\frac{t^2}{4}}$
 $e^{-\frac{t^2}{4}}$

Кор. предел

$f_g(x) \in L_1(\mathbb{R}) \rightarrow y - \text{a.u.}$
 $\text{ад. унр. } \sim \text{н.}$
 $(\frac{1}{i-t})^2, f(t) = \frac{\sin t}{t}$
 $p_g(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} f(t) dt$
 непр. и оц. на

$p_g(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \frac{\sin t}{t} dt = \int_0^{\infty} \cos tx \cdot \frac{\sin t}{t} dt = \dots = p(x)$
 у.н. Карнел
 $\int_0^{\infty} \cos tx = f_g(0) = 1 \text{ у.н. Карнел}$

4.32

$\{S_n\}_{n=1}^{\infty} \text{ c.в. } S_n \sim B(q_n), q_n \rightarrow 0$
 $L_n = q_n S_n, L_n \sim ? (n \rightarrow \infty)$
 $f_{S_n}(t) = \frac{q_n}{1 - (1 - q_n)e^{it}} = \mathbb{E} e^{it S_n}$
 $f_{L_n}(t) = f_{S_n}(q_n t) = f_{L_n}(t)$
 $\lim_{n \rightarrow \infty} \frac{q_n}{1 - (1 - q_n)e^{it q_n}} = \frac{q_n}{1 - (1 + it q_n + o(q_n))}$
 $= \frac{q_n}{-it q_n + q_n + o(q_n)} \rightarrow \frac{1}{1 - it} = \text{хар. гр.}$

не угадываем
 по непрерыв. б
 о сходимости
 непр. и оц. на хар. гр.

Центральная пред. Теорема

$\text{Доп. } \exists S_n, S_n \sim \text{c.в. } \text{с.в. } \text{с.в. } \text{с.в.}$
 $S_n = S_1 + \dots + S_n$

Другие варианты, если $y \sim N(0,1)$, $\sigma = 1$

$Z_n = \frac{S_n - \mathbb{E} S_n}{\sqrt{D S_n}} \rightarrow y \sim N(0,1), \sigma = 1$

$P(Z_n < x) \rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt = \Phi(x)$ на $\mathbb{R}$
 $\text{Th (Леву) } \exists \{S_n\} - \text{н.о.р.с.б. } \subset$
 $\sigma^2 = D S_n \in (0, \infty), \text{ а } \mu = \mathbb{E} S_n$

$\Rightarrow \{S_n\}$ у.нр.н. $\text{с.в. } \text{с.в. } \text{с.в.}$

$P(\frac{S_n - \mu}{\sigma \sqrt{n}} < x) = P(\frac{1}{\sqrt{n}} \sum_{i=1}^n \frac{S_i - \mu}{\sigma} < x) \rightarrow \Phi(x)$
 $\text{с.в. } \text{с.в. } \text{с.в.}$

Д.Т.З. гана (Th (Леву))

$P(b \leq S_n \leq c) \approx P(\frac{b - \mathbb{E} S_n}{\sqrt{D S_n}} \leq \frac{S_n - \mathbb{E} S_n}{\sqrt{D S_n}} \leq \frac{c - \mathbb{E} S_n}{\sqrt{D S_n}}) \approx$
 $\approx \Phi(\frac{c - \mathbb{E} S_n}{\sqrt{D S_n}}) - \Phi(\frac{b - \mathbb{E} S_n}{\sqrt{D S_n}})$

$\frac{1}{\sqrt{n}}$ сходимости, если 3 момента есть

$\exists S_n = \begin{cases} 1, & p \\ 0, & q = 1-p \end{cases} \quad \begin{cases} \text{? } \text{Мульти-Пармаса} \\ \text{бывает} \\ \text{у.нр.н. Леву} \end{cases}$
 $a = p, \sigma^2 = pq$

Другие с.в. с.в. б. Кар. Леву

Б.о.о. $a=0, \sigma=1$

Th (Кар. б. Берну-джен)

б. $\text{с.в. } \text{с.в. } \text{с.в.}$

$P_{n,\delta} := \mathbb{P} \{ \frac{S_n - \mu}{\sigma \sqrt{n}} \leq \delta \}$, где нек. $\delta \in (0,1)$
 $\Rightarrow \Delta_n := \sup_x |P(\frac{S_n - \mu}{\sigma \sqrt{n}} \leq x) - \Phi(x)| \leq \frac{C}{\sqrt{n}} \cdot \frac{P_{n,\delta}}{\delta^{3/2}}$

$C_8 \leq 1.87, \delta \leq 0.469$

$\forall n \in \mathbb{N}$

$C_1 \geq \frac{\sqrt{10}+1}{6\sqrt{2\pi}} = 0.4097$

$P_3 = \mathbb{E} |S_1|^3 = \mathbb{E} \left| \frac{S_1 - \mu}{\sigma} \right|^3 = \frac{p - q^2}{\sigma^3}$

с.в. Бернулли, $C_1 = \frac{\sqrt{10}+1}{6\sqrt{2\pi}}$

4.125

$S_1, S_2, \dots$ - н.о.р.с.б. $\sim N(\mu, \sigma^2)$

$S_n^2 = S_1^2 + \dots + S_n^2$

а) $\lim_{n \rightarrow \infty} P \{ \left| \frac{S_n^2}{n} - 1 \right| > \varepsilon \} = 0 \quad \forall \varepsilon > 0$

$P \{ \left| \frac{S_1^2 + \dots + S_n^2}{n} - 1 \right| > \varepsilon \} \xrightarrow{(*)}$

$\frac{1}{n} \mathbb{E} S_1^2 + \dots + S_n^2 = \frac{1}{n} \mathbb{E} (S_1^2 + \dots + S_n^2) = 1$

$\Rightarrow 0$ (с.в. б. $\text{с.в. } \text{с.в.}$)

3С4
4.125
4.129
4.131
4.136
4.137
4.120
4.121
4.32

$$P\left(\left|\frac{S_n - ES_n}{n}\right| \geq \varepsilon\right) \leq \frac{1}{n^2 \varepsilon^2} DS_n \rightarrow 0$$

$$S) P\left\{\frac{S_n - ES_n}{\sqrt{DS_n}} \leq x\right\} \rightarrow \Phi(x) \text{ as } x \rightarrow \infty$$

$$D(S_n) = ES_n^2 - (ES_n)^2 < \infty$$

$$\Phi(x), -\infty < x < \infty$$

§4.106

$$P\{X_j^{(n)} = \pm n^{1/3}\} = \frac{1}{2} n^{-2/3}$$

$$P\{X_j^{(n)} = 0\} = 1 - n^{-1/3}, ES_j^{(n)} = 0, D_j^{(n)} = 1$$

$$X_1^{(n)}, \dots, X_n^{(n)} \text{ — i.i.d. c.v. } X_n$$

T_n — время до первого успеха. (здесь $n \rightarrow \infty$)

$$S_n = \sum_{j=1}^n X_j^{(n)} = \frac{1}{n} \sum_{j=1}^n X_j^{(n)}, f_{S_n}(t) = \mathbb{E} e^{it S_n} = \mathbb{E} e^{it \sum_{j=1}^n X_j^{(n)}} =$$

$$= (f_{X_1^{(n)}}(t/n))^n, f_{X_1^{(n)}}(t/n) = \mathbb{E} e^{it X_1^{(n)}/n} =$$

$$= \frac{-\cos t/n}{n^{2/3}} + 1 - n^{-1/3} =$$

$$= 1 - n^{-1/3} (1 - \cos t/n)$$

$$\approx (1 - n^{-1/3} (1 - \cos t/n))^n \xrightarrow{n \rightarrow \infty} \frac{e^{-t^2/2}}{2}$$

2/2

§4.129

$X_1, X_2, \dots$ — i.i.d. c.v. $N(0,1)$

$$Z_n = \frac{X_1 + \dots + X_n}{\sqrt{n}} \text{ — случайное блуждание}$$

$$\lim_{n \rightarrow \infty} P\{Z_n \leq x\}, -\infty < x < \infty$$

$$= \lim_{n \rightarrow \infty} (1 - P\{Z_n > x\}) = \lim_{n \rightarrow \infty} \int_0^x \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2}) \sqrt{\pi}} e^{-t^2/2} dt$$

$$\left(1 + \frac{t^2}{n}\right)^{-n/2} dt, \mathbb{E} Z_n = 0, D Z_n = 1$$

Сумма независимых нормальных — нормальная

$$Z_n = \sum_{i=1}^n \frac{1}{\sqrt{n}} X_i = \sum_{i=1}^n \left(\frac{X_i}{\sqrt{n}}\right), \mathbb{E} Z_n = 0$$

$$\mathbb{E} D\left(\frac{X_i}{\sqrt{n}}\right) = \frac{1}{n} \mathbb{E} D X_i = \frac{1}{n} \mathbb{E} X_i^2 = 1$$

$$\mathbb{E} \frac{X_i^2}{n} \leftarrow \frac{1}{n} \sum_{i=1}^n X_i^2 \text{ no } \mathbb{E} X_i^2 = 1$$

§4.121

$$I_1 \approx RC - 95 \cdot 10^{-3}, I_2 \approx 5 \cdot 10^{-3}$$

Решение

$$P\left(\frac{S_n}{\sqrt{n}} < x\right) = \Phi(x)$$

$$\Rightarrow P(S_n < x \cdot \sqrt{n}) = \Phi(x)$$

§4.120

$$Z_n = \frac{S_n - \lambda n}{\sqrt{\lambda n}}$$

$n \rightarrow \infty: S_n \approx \sum_{i=1}^n X_i, X_i \sim \text{Poisson}(\lambda)$

$n = \sum_{i=1}^n 1 = \sum_{i=1}^n \frac{1}{n} \cdot n$

$S_n = \sum_{i=1}^n X_i, \mathbb{E} S_n = \lambda n, D S_n = \lambda n$

$\lim_{n \rightarrow \infty} P\left(\frac{S_n - \lambda n}{\sqrt{\lambda n}} \leq x\right) = \Phi(x)$

$S_{n+1} = S_n + X_{n+1}$ — случайное блуждание

§4.132

$$X_1 \sim \text{Poisson}(\lambda); \lim_{\lambda \rightarrow \infty} P\left\{\frac{X_1 - \lambda}{\sqrt{\lambda}} \leq x\right\} = \Phi(x)$$

$$f_X(t) = \mathbb{E} e^{itX_1} = e^{it\lambda} f_X\left(\frac{t}{\lambda}\right)$$

$$e^{it\lambda} e^{it(\frac{t}{\lambda} - 1)} \rightarrow e^{-\frac{t^2}{2}} N(0,1)$$

$$\lim_{\lambda \rightarrow \infty} P(-) = \Phi(x)$$

§4.137

Условие экв. 9.

кн. работа.

$$\mathbb{E} \frac{X_i^2}{n} = 1; \mathbb{E} \frac{X_1^2 + \dots + X_n^2}{n} = \mathbb{E} X_i^2 = 1$$

Дана $X = (X_1, \dots, X_n)$ — выборка, если $X_1, X_2, \dots, X_n$ — независимы на $(\Omega, \mathcal{F}, P)$ с.н.

4.2,
4.1
Werner
Meerbach

Дистрибуция	Распр. / формула	E	D	Хар-а функ.
1) $\xi = a, a = \text{const}$	$P(\xi = a) = 1$	a	0	e^{ita}
2) $B(n, p)$ $n \in \mathbb{N}, 0 \leq p \leq 1$	$P(\xi = k) = \binom{n}{k} p^k (1-p)^{n-k}$ $k = 0, 1, \dots, n$	np	$np(1-p)$	$(pe^{it} + 1 - p)^n$
3) $G(p)$ $0 < p \leq 1, q = 1-p$	$P(\xi = k) = p(1-p)^k$ $k = 0, 1, \dots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-qe^{it}}$
4) $NB(r, p)$ $r \geq 2$ целая $r > 0, p \in (0, 1)$ Бернулли-геом.	$P(\xi = k) = \binom{k+r-1}{k} p^r (1-p)^k$ $k = 0, 1, \dots$	$\frac{rp}{1-p}$	$\frac{rp}{(1-p)^2}$	$\left(\frac{1-p}{1-pe^{it}}\right)^r$
5) $Pois(\lambda)$ $\lambda \in (0, \infty), k \in \mathbb{N}_0$	$P(\xi = k) = \frac{\lambda^k}{k!} e^{-\lambda}$	λ	λ	$e^{\lambda(e^{it}-1)}$
6) $N(\mu, \sigma^2)$ $\sigma^2 > 0$	$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2	$e^{it\mu - \frac{\sigma^2 t^2}{2}}$
7) $\exp(\lambda)$ $\lambda > 0, x \in [0, \infty)$	$p(x) = \begin{cases} \lambda e^{-\lambda x}, & x \in [0, \infty) \\ 0, & x \in (-\infty, 0) \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - it}$
8) $\Gamma(\alpha, \lambda)$ $\lambda > 0, \alpha > 0$ $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$	$p(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left(\frac{\lambda}{\lambda - it}\right)^\alpha$
9) $L(\alpha, \beta)$ $\alpha > 0, \beta \in \mathbb{R}$	$p(x) = \frac{\alpha}{2} e^{-\alpha x-\beta }$	β	$\frac{2}{\alpha^2}$	$\frac{e^{\beta it}}{1 + \frac{t^2}{\alpha^2}}$
10) $K(a, b)$ $a \in \mathbb{R}, b > 0$	$p(x) = \frac{1}{\pi} \left[\frac{b}{(x-a)^2 + b^2} \right]$	$\mathbb{R}$	$\mathbb{R}$	$e^{ait - b t }$
11) $R(a, b)$ $a, b \in (-\infty, +\infty)$	$p(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{иначе} \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{itb} - e^{ita}}{it(b-a)}$
12) $St(n)$ $n \in \mathbb{R}, n > 0$	$p(x) = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2}) \sqrt{\pi}} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$	$0, n > 1$ $\mathbb{R}, n = 1$	0	

- 1) $N(\theta_1, \theta_2^2)$ - ДУ ген θ_1
 θ_2 - изв. $G(X, \theta_1) = \frac{1}{\theta_2} (\bar{X} - \theta_1) \sim N(0, 1) \sim \chi_0$
 θ_2 - неизв. $G(X, \theta_1) = \frac{1}{\theta_2} (\bar{X} - \theta_1) \sim t_{n-1}$
- 2) $N(a, \sigma^2)$ - ДУ ген σ^2
 σ^2 - изв. $G(X, \sigma^2) = \frac{1}{\sigma^2} \sum_{i=1}^n (X_i - \bar{X})^2 \sim \chi^2(n)$
 σ^2 - неизв. $G(X, \sigma^2) = \frac{1}{\sigma^2} \sum_{i=1}^n (X_i - \bar{X})^2 \sim \chi^2(n-1)$